The Thirty-Sixth Annual
Eastern Shore High School Mathematics Competition

November 14, 2019

Team Contest Exam

Instructions

Answer as many questions as possible in the time provided. To receive full credit for a correct solution, show all work and provide a clearly written explanation. Solutions will be judged based on correctness, completeness and clarity. (Little credit, if any, will be given for a solution consisting of just a number or a single sentence.) Calculators are allowed **only** on the team contest exam.

All work and answers must be written on the provided sheets of plain white paper. Use only one side of each sheet of paper, and start each new problem on a new sheet of paper. Write the name of the school which you are representing at the top of each sheet that you turn in for scoring.

At the start of the team round, your team will receive a copy of only Problem 1. Your team must submit a response to Problem 1 within the first 15 minutes of the team round time interval.

When you submit your response for Problem 1, you will receive a copy of Problem 2 and a copy of Problem 3. Your team will then have the time remaining in the team round to complete a response for each problem.

Note: if your team completes Problem 1 before the end of the allotted time, you may submit it and receive copies of Problem 2 and Problem 3 in advance.

1. Two such rectangles are possible. Let X and Y represent the respective dimensions of a rectangle. Since the dimensions of the rectangle are both positive integers it follows that the total number of square units in the rectangle, T, is equal to XY and one half of the total number of square units in the rectangle, T, is equal to the number of square units in the border of the rectangle.

$$\frac{1}{2}(T) = 2X + 2Y - 4$$

$$\frac{1}{2}(XY) = 2X + 2Y - 4$$

$$XY = 4X + 4Y - 8$$

$$XY - 4Y = 4X - 8$$

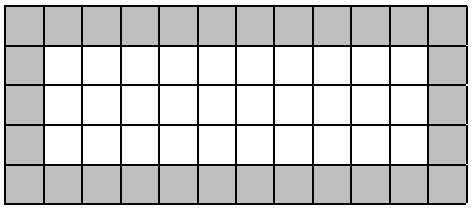
$$Y(X - 4) = 4X - 8$$

$$Y = \frac{(4X - 8)}{(X - 4)}$$

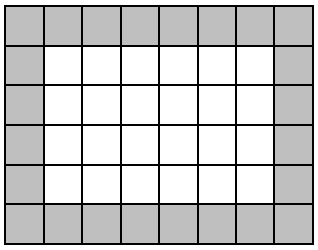
You may either graph the equation of $Y = \frac{(4X-8)}{(X-4)}$ and identify the possible positive integral values for X and Y or construct a "table" of values for the graph of $Y = \frac{(4X-8)}{(X-4)}$ with a "table start" at 1 and a "delta table" setting of 1 and identify the possible positive integral values for X and Y when $0 < XY \leq 100$.

The dimensions of the two rectangles are 5 × 12 and 6 × 8 .

5 × 12



6 × 8

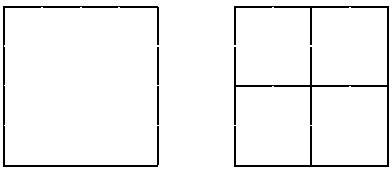


Reference:

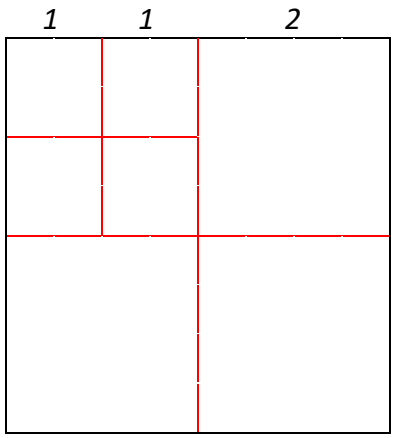
Number Freak: From 1-200, The Hidden Language of Numbers Revealed, by Derrick Niederman (2009)

2. (Other solutions may be possible.)

A 4-unit square can be cut into exactly 4 square pieces with none of the original square left over. As shown below each of the 4 square pieces is a 1-unit square.

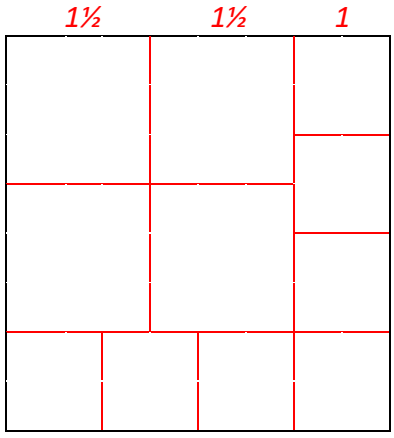


Part 1: A 4-unit square can also be cut into exactly 7 square pieces with no part of the original 4-unit square left over. However, not all of the squares are congruent. Clearly indicate how the 4-unit square shown below can be cut into exactly 7 square pieces with none of the original 4-unit square left over AND provide a summary of the dimensions of the 7 smaller squares.



Summary: ***FOUR 1-UNIT*** *squares and* ***THREE 2-UNIT*** *squares.*

Part 2: A 4-unit square can also be cut into exactly 11 square pieces with none of the original 4-unit square left over. However, not all of the squares are congruent. Clearly indicate how the 4-unit square shown below can be cut into exactly 11 square pieces with none of the original 4-unit square left over AND provide a summary of the dimensions of the 11 smaller squares.



Summary: ***FOUR 1½ -UNIT*** *squares and* ***SEVEN 1-UNIT*** *squares.*

3.

(a) The largest total that can be made is 12(10) + 12(9) = 228, where all 24 tiles are drawn. 12(10) + 11(9) = 219 is the next largest total that can be formed, drawing 23 tiles. The next largest is 11(10) + 12(9) = 218, also drawing 23 tiles.

(b) 227 cannot be formed, since the 228 is the largest possible total and 219 is the second largest.