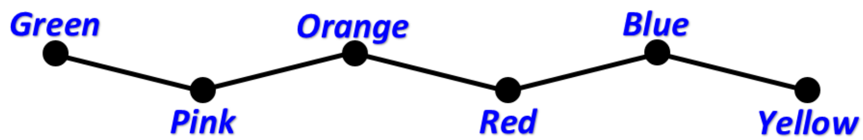
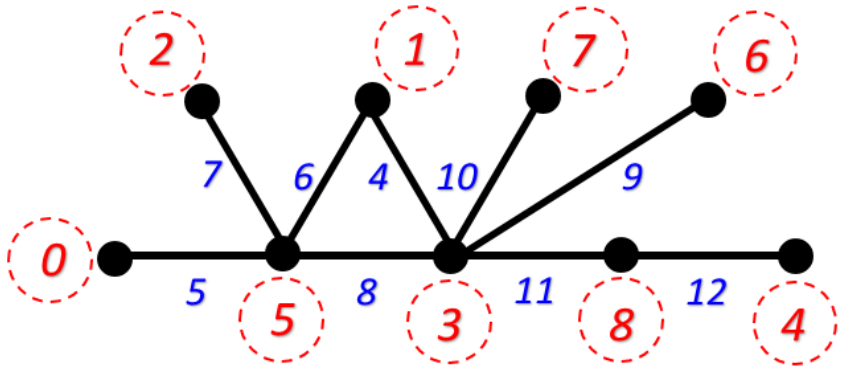
1. The following diagram could also be drawn with the colors in reverse order.



2.



3. Let $A(r)$ be the area of the black region when the outermost circle is radius r. Then the area of the black region is given by

$$A(r) = \pi r^2 - r^2 \frac{3}{2}\sin(\pi/3) + A(r/2) = \pi r^2 - r^2 \frac{3\sqrt{3}}{4} + A(r/2)$$

Unrolling the recurrence relation, we end with

$$A(r) = r^2 \left(\pi - \frac{3\sqrt{3}}{4}\right) \sum_{k=0}^{\infty} \left(\frac{1}{2^k}\right)^2 = r^2 \left(\pi - \frac{3\sqrt{3}}{4}\right)\left(\frac{1}{1-1/4}\right) = r^2 \left(\pi - \frac{3\sqrt{3}}{4}\right)\left(\frac{4}{3}\right)$$

So,

$$A(r) = \frac{4\pi r^2}{3} - r^2\sqrt{3}$$

The area of the circle of radius r is πr^2, so our probability is

$$\frac{A(r)}{\pi r^2} = \frac{4}{3} - \frac{\sqrt{3}}{\pi}$$

Alternatively, we can note that since we are in 2-dimensions, and a scaling of the radius scales by a square factor. So,

$$A(r) = \pi r^2 - r^2 \frac{3\sqrt{3}}{4} + (1/4)A(r)$$

Simplifying, we reach the same value for $A(r)$:

$$A(r) = \frac{4}{3}\left(\pi r^2 - r^2 \frac{3\sqrt{3}}{4}\right) = \frac{4\pi r^2}{3} - r^2\sqrt{3}$$

As before, we find that the probability is

$$\frac{A(r)}{\pi r^2} = \frac{4}{3} - \frac{\sqrt{3}}{\pi}.$$