

1. **Part 1:** The solutions are:

$$235 + 746 = 981$$

$$236 + 745 = 981$$

$$245 + 736 = 981$$

$$246 + 735 = 981$$

$$324 + 657 = 981$$

$$327 + 654 = 981$$

$$354 + 627 = 981$$

$$357 + 624 = 981$$

Part 2: : The solutions are:

$$243 + 576 = 819$$

$$246 + 573 = 819$$

$$273 + 546 = 819$$

$$276 + 543 = 819$$

$$352 + 467 = 819$$

$$357 + 462 = 819$$

$$362 + 457 = 819$$

$$367 + 452 = 819$$

$$234 + 657 = 891$$

$$237 + 654 = 891$$

$$254 + 637 = 891$$

$$257 + 634 = 891$$

$$324 + 567 = 891$$

$$327 + 564 = 891$$

$$364 + 527 = 891$$

$$367 + 524 = 891$$

$$243 + 675 = 918$$

$$245 + 673 = 918$$

$$273 + 645 = 918$$

$$275 + 643 = 918$$

$$342 + 576 = 918$$

$$346 + 572 = 918$$

$$372 + 546 = 918$$

$$376 + 542 = 918$$

2. (a) If $GHI = 345$, then it must be true that $\{A, D\} = \{1, 2\}$, leaving only 6, 7, 8, and 9 for the remaining digits. This forces $\{C, F\}$ to be either $\{6, 9\}$ or $\{7, 8\}$. If $\{C, F\} = \{6, 9\}$, then $\{B, E\} = \{7, 8\}$, which cannot be the case because that would make $H = 6$. If $\{C, F\} = \{7, 8\}$, then $\{B, E\} = \{6, 9\}$, which would again make $H = 6$. However, we know that $H = 4$. Thus, GHI cannot equal 345.
- (b) Suppose $GHI = 548$. This leaves 1, 2, 3, 6, 7, and 9 for the remaining digits. It follows that either $\{C, F\} = \{1, 7\}$ or $\{C, F\} = \{2, 6\}$. In the first case, we are unable to choose B and E so that H will equal 4, so we must have $\{C, F\} = \{2, 6\}$. We must then have $\{B, E\} = \{1, 3\}$ since only 1, 3, 7, and 9 remain. This means that $\{A, D\} = \{7, 9\}$ which would cause the sum to be a 4-digit number. Thus, GHI cannot equal 548.

3. (a) $4 \times 13 = 52$ is the only solution.

- (b) Suppose $a \times b = c$ is the equation we are seeking. First, notice that neither a nor b can end in 5, as then c would end in 0 or 5, which is not allowed. Similarly, if either a or b ended in 1, we would need to reuse a digit for the ones place of c . We would have a similar situation if either a or b ended in 6.

Additionally, notice that if one of our factors a and b is a 1-digit number, the other factor must have two digits. That is, our factors a and b cannot be a 1-digit and a 3-digit number, because then c would have three digits, but we only have six digits total with which to work. Clearly, two 1-digit numbers will not work, nor will a 1-digit and a 4-digit (or 5-digit) number. Similarly, if one of a and b is a 2-digit number, the other factor must have one digit because if it has more, the number of digits in c will push the total number of digits over six.

Hence, the only possibility is that one of our factors has one digit and the other two digits. Consequently, c must have three digits. For simplicity, assume that a has one digit and b has two.

Since neither factor can end in 1, 5, or 6, the only possibilities for a are 2, 3, and 4.

If a is 2, b must end in either 3 or 4. b must also be at least 50 since c must have three digits. One can easily check that the only possible values of b are 53, 63, 54, and 64, but none of these will work.

If a is 4, b must end in either 2 or 3. b must be at least 25 since c must have three digits. The only possible values of b are therefore 32, 52, 62, 53, and 63, but none of these will work.

Thus, if a solution exists, $a = 3$. In this case, b must end in either 2 or 4 and must be greater than 33. The only possible values of b are 42, 52, 62, 54, and 64.

One can check that the only solution is therefore $3 \times 54 = 162$.