The Thirty-Third Annual
Eastern Shore High School Mathematics Competition

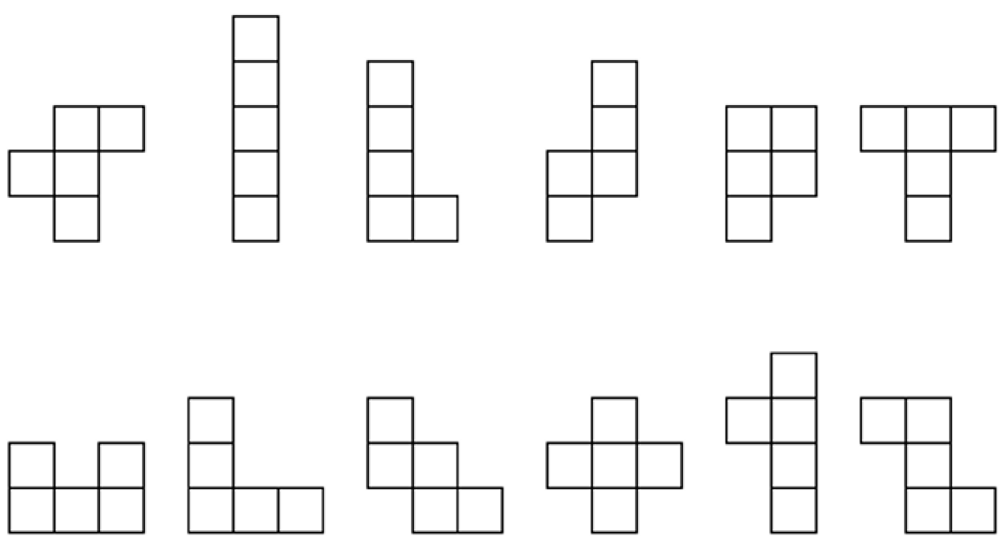
November 10, 2016

Team Contest Exam Solutions

1. PART 1: The numbers for hexominoes that can be folded into a cube are highlighted in the following figure.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35

PART II: There are 12 free pentominoes, as shown in the following figure.



2. a. There are 5 squares in the 2×2 grid: one 2×2 square and four 1×1 squares.

   b. There are 14 squares in the 3×3 grid: one 3×3 square, four 2×2 squares, and nine 1×1 squares.

   c. There are 30 squares in the 4×4 grid and 55 squares in the 5×5 grid. Notice that, in general for an $n \times n$ grid, there will be

   one $n \times n$ square
   four $(n-1) \times (n-1)$ squares
   nine $(n-2) \times (n-2)$ squares
   ...
   $(n-1)^2$ 2×2 squares
   n^2 1×1 squares

   So, the total number of squares in an $n \times n$ grid is $1^2 + 2^2 + \cdots + n^2$. This expression is also equal to $\dfrac{n(n+1)(2n+1)}{6}$.

3. The area of the rectangle must be the sum of the squares of the given side lengths, 1056. Since 18 is the side length of one of our given squares, both sides of the rectangle must be at least 18.

The only possible dimensions for our rectangle are then found to be 32×33, 24×44, or 22×48 by examining factors of 1056.

24×44 and 22×48 can be ruled out since the square with side length 18 cannot be placed in these cases, and we conclude that the dimensions of the rectangle must be 32×33. The following diagram shows a 32×33 rectangle satisfying the given conditions.