The Thirty-First Annual
Eastern Shore High School Mathematics Competition

November 6, 2014

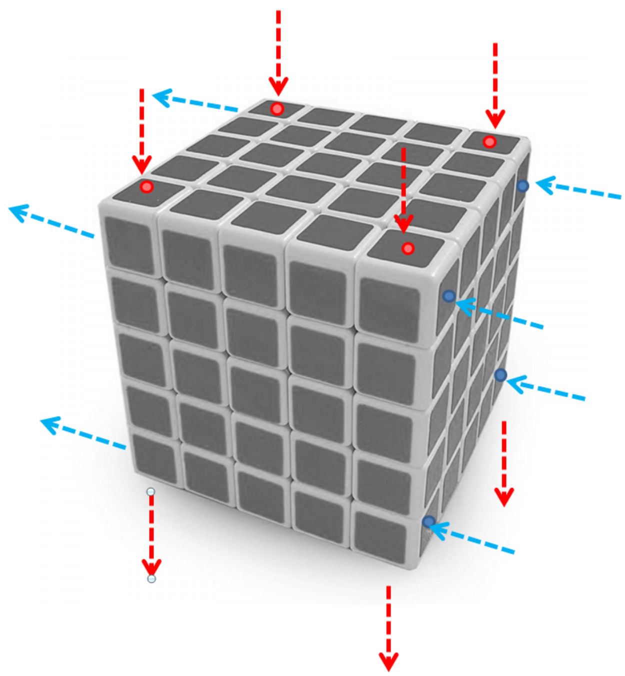
Team Contest Exam Solutions

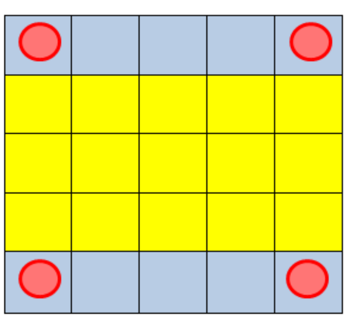
1. One hundred twenty-five congruent cubes have been used to form a "big cube" that is solid. You drill the center of each corner cube of the side that faces you and continue to drill all the way through to the other side of the "big cube." You repeat the process starting on the top side of the "big cube." After you have finished drilling the top you examine the 125 cubes that formed the "big cube," how many of those cubes do not have a whole drilled through them? Provide appropriate justification for your answer.

   Note: you have been given 30 cubes to possibly assist you.

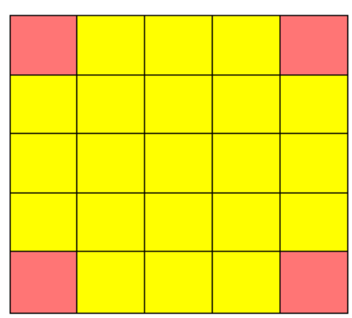
   Solution:

   The following figures show how to find the answer of 93 undrilled cubes.





Top AND Bottom Layers of the Cube have 15 undrilled cubes in each layer [30]



Three Inner Layers of the Cube have 21 undrilled cubes in each layer [63]

30 + 63 = 93 Total Undrilled Cubes.

2. a. Provide justification that 541 is a prime number.

   b. Find a prime number larger than 541. Provide justification that the number is a prime number.

   Solution:

   a. $\sqrt{541} \approx 23.3$, so we only need to verify that 541 is not divisible by any prime less than or equal to 23.

   b. Answers can be verified using the same divisibility test from part a.

3. A **palindrome** is a number that is the same when written forwards or backwards. For example, 12321 is a palindrome.

   Find two 3-digit palindromes that are also prime numbers and differ by 10. Provide justification that each is a prime number.

   Solution: The 3-digit palidromic primes are 101, 131, 151, 181, 191, 313, 353, 373, 383, 727, 757, 787, 797, 919, and 929. The possible solutions are then

   191 and 181
   383 and 373
   797 and 787
   929 and 919

   To find palindromic primes, one could find all 3-digit palindromes and check whether each is prime. However, we do not need to check all of the 3-digit palindromes. We can eliminate any palindromes ending in 2, 4, 6, or 8 as then we would have an even 3-digit number. Also, 0 cannot be the last (and first) digit since we must have a 3-digit number. The palindrome cannot end in 5 either, as then the 3-digit number would be divisible by 5.

   Thus, our palindrome must end (and start) with either 1, 3, 7, or 9. The digits cannot all be the same because then the palindrome would be divisible by 3. So, there are 4 possibilities for the last digit, 9 possibilities for the second digit once the first has been chosen, and only 1 possibility for the last digit (i.e., the same as the first digit).

   By using the divisibility test from problem 2, we can find that 191 and 181 comprise the smallest pair of 3-digit palindromic primes that differ by 10. The other solutions can be found similarly.