The Twenty-Eighth Annual

Eastern Shore High School Mathematics Competition

November 10, 2011

Individual Contest Exam

Instructions

There are twenty problems on this exam. Select the best answer for each problem.

Your score will be the number of *correct* answers that you select.
There is no penalty for incorrect answers.

The use of a calculator is *not* permitted on this exam.

In the event of tie scores, #18, #19 and #20 will be used as tiebreakers.

1. A father is currently four times as old as his daughter. In six years, he will be three times as old as she will be. How old was the father when the daughter was born?

   (a) 24 (b) 36 (c) 48 (d) 54 (e) Not enough information is given

2. The Empirical Rule states that for data whose frequency distribution has a mound-shaped and symmetric graph, approximately 95% of the observations will lie within 2 standard deviations around the mean. Mrs. Jones eleventh grade math class took a test. The mean score was 78 and the standard deviation was 4.5. Assuming the graph of the scores had a mound shape, which statement below is correct?

   (a) Approximately 95% of the scores are between 76 and 80.
   (b) Approximately 95% of the scores are between 73.5 and 82.5.
   (c) Approximately 95% of the scores are between 69 and 87.
   (d) All the scores are between 70 and 79.
   (e) Approximately 95% of the scores are above 78.

3. Let $g(x) = \dfrac{2x+1}{x+2}$. Find the value of $(g \circ g)(2)$.

   (a) 4/5 (b) 14/13 (c) 5/4 (d) 25/16 (e) 2

4. The heights, in inches, of a group of high school seniors were measured and are statistically described below.

Mean= 68 in.	First quartile= 63 in.
Mode= 70 in.	Third quartile= 71 in.
Median = 66.5 in.	

   Consider the following statements:

   I. 50% of the seniors are more than 68 inches tall.

   II. 25% of the seniors are less than 63 inches tall.

   III. The most frequent height is 70 inches tall.

   IV. The middle 50% of the seniors are between 63 and 71 inches tall.

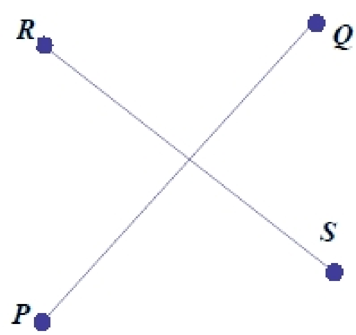
   Which of the above statements must be true?

   (a) All of the statements are true. (b) Only statements II, III, and IV are true.
   (c) Only statement I is true. (d) Only statement III is true.
   (e) None of the statements is true.

5. If $f(x) = 2x^3 - 5$, what is the formula for the inverse function $f^{-1}(x)$?

(a) $2\sqrt[3]{x} - 5$ (b) $\sqrt[3]{\dfrac{x+5}{2}}$ (c) $\dfrac{\sqrt[3]{x}}{2} + 5$ (d) $\sqrt[3]{2x^3 - 5}$ (e) $\dfrac{x^{-3}}{2} + 5$

6. In the following diagram, which is not necessarily drawn to scale, $\overline{\text{PQ}}$ is perpendicular to $\overline{\text{RS}}$. The coordinates of P are (1, 1), the coordinates of Q are (5, 3), and the coordinates of R are (2, 4). If the y-coordinate of S is 2, what is the x-coordinate of S?



(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

7. Tyler was a generous guy. One month, he cashed his paycheck and started to distribute the money to his friends. The first friend received 1/3 of the money, the second friend received 1/3 of the remaining money, the third friend received 1/3 of the remaining money left after the first and second friend, the fourth friend received 1/3 of what remained after the friends before him received their share... and so on. If Tyler continued to give out his money in this manner, what fraction of Tyler's paycheck did the n^{th} friend receive?

(a) $\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^n$ (b) $\left(\frac{1}{3}\right)^n$ (c) $\left(\frac{1}{3}\right)^{n-1}$ (d) $\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{n-1}$ (e) $\left(\frac{2}{3}\right)^n$

8. Tomorrow's date, written in the standard mm/dd/yy format, is 11/11/11. This date will have the peculiar property that the numbers representing the month (November is the 11th month of the year), day of the month (11), and year (2011), are all the same. The next date with this property will be 12/12/12 (on December 12, 2012).

From January 1, 2013 through December 31, 2999, how many dates will have this property?

(a) less than 120 (b) exactly 120 (c) more than 120, but less than 1200
(d) exactly 1200 (e) more than 1200

9. In a 1-mile race you run the first half in 4 minutes, the next half of the remaining portion of the race in twice the time, the next half of the remaining portion in four times the time of the first portion, the next half of the remaining portion in eight times the time of the first portion, and so on. How far have you travelled after 28 minutes?

   (a) $\frac{3}{4}$ mile (b) $\frac{7}{8}$ mile (c) $\frac{15}{16}$ mile (d) $\dfrac{2^{28}-1}{2^{28}}$ miles

   (e) 1 mile and now you are sitting happily at the finish line

10. An urn is filled with coins and beads, each of which is either silver or gold. Twenty percent of the objects in the urn are beads. Forty percent of the coins in the urn are silver. What percentage of the objects in the urn are gold coins?

    (a) 40% (b) 48% (c) 60% (d) 75% (e) 92%

11. How many integers between 1 and 100 have exactly three distinct factors?

    (a) 3 (b) 4 (c) 5 (d) 9 (e) 10

12. Let $F_n(x)$ denote the n^{th} digit after the decimal point in the decimal representation of x. For example, since $\frac{3}{8} = 0.375$, we would write $F_1\left(\frac{3}{8}\right) = 3$, $F_2\left(\frac{3}{8}\right) = 7$, and $F_3\left(\frac{3}{8}\right) = 5$. Find the value of $F_{28}\left(\frac{5}{28}\right)$.

    (a) 0 (b) 1 (c) 5 (d) 7 (e) 8

13. Which of the following statements are logically equivalent?

    I. All griblers grable.

    II. Anything that does not grable is not a gribler.

    III. If it grables, it's a gribler.

    IV. If it doesn't grable, it is not a gribler.

    V. If it is not a gribler, it doesn't grable.

    (a) All of them are equivalent. (b) I, II and IV are equivalent.
    (c) I and III are equivalent. (d) IV and V are equivalent.
    (e) III and IV are equivalent.

14. Which of the following is equal to

$$\sqrt[3]{\sqrt{52}-5} \cdot \sqrt[3]{\sqrt{52}+5}?$$

    (a) 3 (b) $\sqrt[3]{77}$ (c) $\sqrt[3]{104}$ (d) $\sqrt[3]{47 \cdot 57}$ (e) 47

15. If $y = a \cdot \sin(\omega t + b)$ describes the location of a point in simple harmonic motion, then the velocity of the point at any time t is given by $v = a\omega \cdot \cos(\omega t + b)$. For what values of t (in terms of b and ω) is the velocity a maximum? (In each of the following, k represents any integer.)

(a) $t = \dfrac{-b + 2k\pi}{\omega}$ (b) $t = \dfrac{-b + (2k+1)\pi}{\omega}$ (c) $t = \dfrac{-\omega + 2k\pi}{b}$ (d) $t = \dfrac{-\omega + (2k+1)\pi}{b}$

16. Let S denote the solution set of the equation $6x^2 - 212 - 4\sqrt{6x^2 - 212} + 4 = 0$.
What is the *product* of all of the elements of S?

(a) -36 (b) -$\frac{106}{3}$ (c) -4 (d) 0 (e) $\frac{106}{3}$

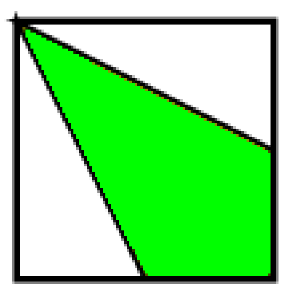
17. Find the solution of the equation $5^{2x+1} = 6^{x-2}$.

(a) -17/4 (b) -3 (c) $\dfrac{-1 - 2\log_5 6}{2 - \log_5 6}$ (d) $\dfrac{-2 - \log_5 6}{1 - 2\log_5 6}$ (e) $\dfrac{1 - 2\log_6 5}{-2 - 2\log_6 5}$

18. (Tiebreaker #1) Let A_n denote the set of positive integer multiples of n that are less than or equal to 1000. How many elements are in the set $A_2 \cup A_3$?

(a) 166 (b) 200 (c) 333 (d) 667 (e) 833

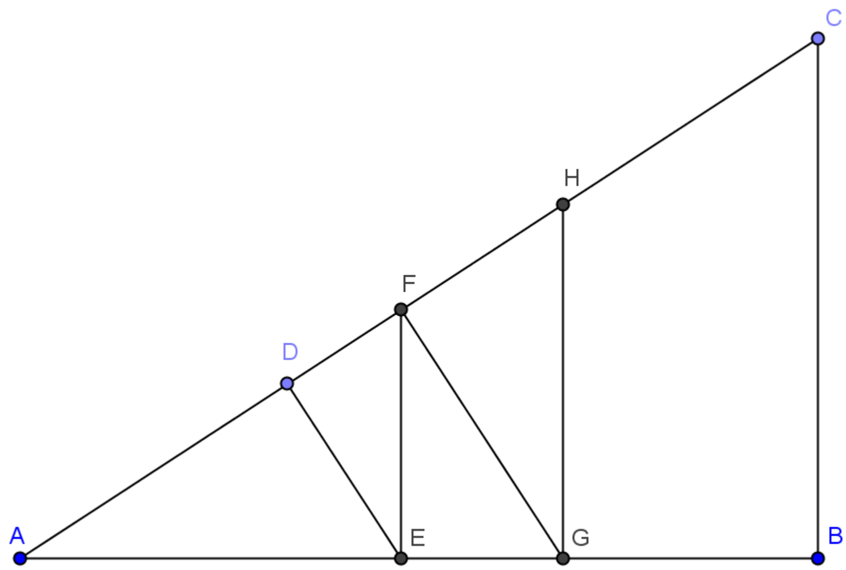
19. (Tiebreaker #2) When an interior angle of a square with area of 100 square units is trisected, the figure shown below is created.



(Note that this figure is not necessarily drawn to scale.) If A is the area of the shaded region, then which of the following is true about A?

(a) $38 < A < 41$ (b) $41 < A < 44$ (c) $44 < A < 47$ (d) $47 < A < 50$ (e) $A \geq 50$

20. (Tiebreaker #3) In the image below (which is not necessarily drawn to scale), triangle $\triangle ABC$ is a right triangle, lines DE and FG are perpendicular to AC and lines EF and GH are perpendicular to AB. If the length of AE is 8, the length of GB is 6, and the length of FH is $\sqrt{5}$, find the length of BC.



(a) 5 (b) 6 (c) 7 (d) 8 (e) None of these