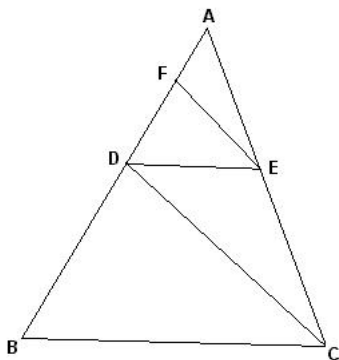


Eastern Shore High School Mathematics Competition
November 12, 2003
Individual Contest

Choose the best response for each of the following questions.

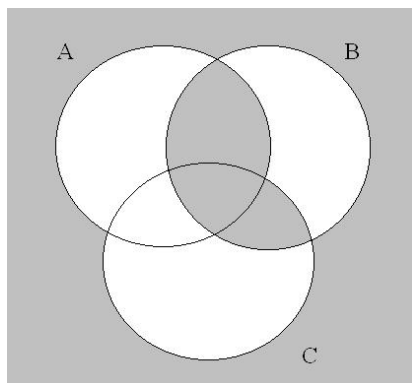
1. For the arithmetic progression $1, 4, 7, 10, 13, \dots$, what is the sum of the first 50 terms?
(a) 148×25 (b) 3×148 (c) 5×180 (d) 5×745 (e) None of these
2. What is a certain number if $1/2$ of it plus $2/3$ of it is A ?
(a) $\frac{7A}{6}$ (b) $\frac{6}{7A}$ (c) $\frac{7}{6A}$ (d) $\frac{6A}{7}$ (e) None of these
3. Find the area of the triangle bounded by the lines $y = 0$, $y = x$ and $x + 3y = 12$.
(a) 18 (b) 6 (c) 36 (d) 12 (e) None of these
4. If $\log_2 \log_3 \log_4 \log_5 A = x$, then the value of A is
(a) 120^x (b) 2^{60x} (c) $2^{3^{4^{5x}}}$ (d) $5^{4^{3^{2x}}}$ (e) None of these
5. A fair six-sided die is rolled five times. To the nearest thousandth, what is the probability of rolling a “6” at least once?
(a) 0.598 (b) 0.833 (c) 0.167 (d) 0.402 (e) 1.000
6. Consider the sequence $3, 4, 7, 11, 18, \dots$, in which each term (starting with 7) is the sum of the two preceding terms. What digit is in the ones place of the 2003rd term of this sequence?
(a) 1 (b) 2 (c) 3 (d) 4 (e) None of these
7. Which of the following is the ternary (base 3) representation of the decimal number 80?
(a) 24_3 (b) 220_3 (c) 1010000_3 (d) 2222_3 (e) None of these
8. Which of the following positive integers has exactly 1000 distinct factors?
(a) $2^{250} \cdot 7^4$ (b) $3^{500} \cdot 3^2$ (c) $4^{998} \cdot 3^2$ (d) $4^2 \cdot 10^3$ (e) None of these
9. Solve for all real values of x : $\frac{2x-3}{x} > \frac{x+5}{x}$.
(a) $\{x : 0 < x < 8\}$ (b) $\{x : x > 8 \text{ or } x < 0\}$ (c) $\{x : 0 \leq x \leq 8\}$
(d) $\{x : x > 8\}$ (e) None of these

10. In $\triangle ABC$, $DE \parallel BC$, $FE \parallel DC$, $AF = 4$, and $FD = 6$. Find DB .



- (a) 8 (b) 10 (c) 11 (d) 13 (e) 15
11. The shaded region in the Venn diagram below represents which of the following sets? (Choose the correct response from among $\{a, b, c, d, e\}$ below.)

- i. $(A \cap B) \cup (B \cap C) \cup (A \cup B \cup C)'$
 ii. $B \cap (A \cup C) \cup (A' \cap B' \cap C')$
 iii. $((A \cup C) \cap B') \cup (B \cap (A \cup C)')$



- (a) Only set i (b) Only set ii (c) Only set iii (d) All three sets (e) None of these
12. Suppose you are a payroll manager for a firm that employs 150 people. Data from the first week of June last year indicates that the median wage was \$425, the mean was \$535, and the mode was \$415.

By June of this year, each employee will have received a 5% raise over his/her wage in June of last year. How much money is needed to pay these 150 employees this June?

- (a) \$80250 (b) \$84262.50 (c) \$65362.50 (d) \$66937.50 (e) Cannot be determined

13. Suppose $\log_{32} p = q$ and $\log_2 r = q$ for positive numbers p, q and r . Which of the following must be true? (Choose the correct response from among {a,b,c,d,e} below.)

- i. $5 \log_2 r = \log_2 p$
- ii. $p = \frac{q}{5}$
- iii. $5 \log_{32} r = \log_2 p$

- (a) Only (i) must be true
- (b) Only (ii) must be true
- (c) Only (iii) must be true
- (d) All three statements must be true
- (e) None of the statements must be true

14. For each real constant B , how many real solutions (x, y) does the following system have?

$$\begin{aligned} y &= (x - 2)^2 + B \\ y &= B \end{aligned}$$

- (a) 0 (b) 1 (c) 2 (d) Infinitely many
- (e) Not enough information is given: the answer depends on B

15. If $x = \frac{1}{4 + \left(\frac{1}{4 + \frac{1}{4 + \frac{1}{4 + \dots}}} \right)}$, then what is the value of $x + 2$?

- (a) $\pi - 1$ (b) $\sqrt{5}$ (c) $\pm\sqrt{5}$ (d) $\frac{9}{4}$ (e) Cannot be determined

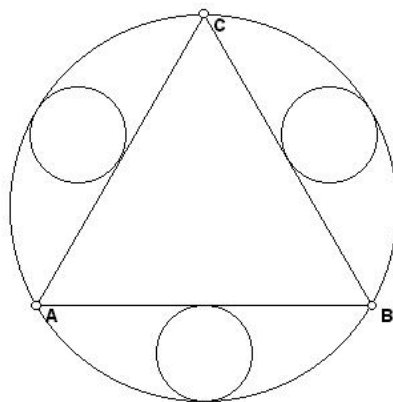
16. Suppose $5x + 3y = 100$, where x and y are both integers. Which of the following, if any, *must* be true?

- (a) y is a positive multiple of 3
- (b) y is a positive multiple of 5
- (c) y is a multiple of 5
- (d) y is a multiple of 3
- (e) None of these must be true

17. How many pairs of positive integers x, y satisfy the conditions $x > y$ and $5x + 3y = 100$?

- (a) two (b) three (c) four (d) infinitely many (e) None of these

18. In the xy -plane, what is the midpoint of the line segment from $(2,3)$ to $(-1,-2)$?
- (a) $(5/2,-3/2)$ (b) $(0,1)$ (c) $(1,1)$ (d) $(1/2,1/2)$ (e) None of these
19. If the statement “*All cyclic groups are Abelian*” is true, then which of the following statements must also be true? (Choose the correct response from among $\{a,b,c,d,e\}$ below.)
- All Abelian groups are cyclic*
 - No group that is not Abelian is cyclic*
 - No group that is not cyclic is Abelian*
- (a) Only (i) must be true
 (b) Only (ii) must be true
 (c) Only (iii) must be true
 (d) All three statements must be true
 (e) None of these statements must be true
20. The triangle $\triangle ABC$ is an equilateral triangle. The three small circles are the largest circles that will fit inside of the big circle and outside of the triangle. If the radius of the large circle is R , what is the combined area of the three smaller circles?



- (a) $\frac{3}{4}(R-2)^2\pi$
 (b) $\frac{3}{7}\left(\frac{R}{5}\right)^2\pi$
 (c) $\frac{3}{R-4}(\pi R)^2$
 (d) $\frac{3}{16}R^2\pi$
 (e) None of the above