

## Introduction

Students are often afraid to solve mathematical word problems (Nosegbe-Okoka, 2004). All too often, panic sets in, self-esteem lowers, thinking shuts down, and an explosion of hands go up in the air pleading for help (Ponce & Garrison, 2005). As a result, students gloss over details of word problems, simply guess what to do, and perform random calculations without logical reasoning (Braddock-Hunt, 2015). How can teachers alleviate students' fears of word problems and ensure that students are conceptualizing this mathematical process?

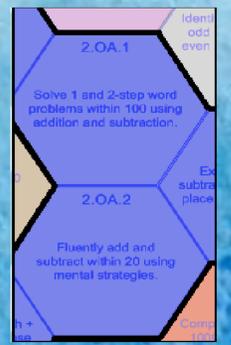
The purpose of our study was to investigate how a group of students entering third grade understand and solve word problems through a sense-making approach with the help of visualization and tactile strategies. Using a behavior and error analysis process, we designed an instructional sequence to better understand where students struggle when solving word problems so that successful strategies and solutions can be acquired.

## References

Barlow, A. (2010). Building word problems: What does it take? *Teaching Children Mathematics*, 17(3), 140-148.  
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 Ponce, G. A., & Garrison, L. (2005). Overcoming the "walls" surrounding word problems. *Teaching Children Mathematics*, 5(2), 256-262.  
 Nosegbe-Okoka, C. (2004). A sense-making approach to word problems. *Mathematics Teaching in the Middle School*, 10(1), 43-45.  
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## Literature Review

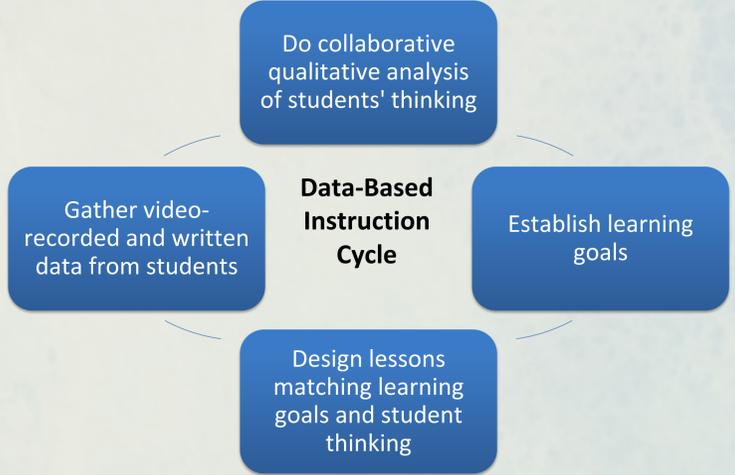
Literature shows there are several strategies teachers can use to help students with word problems in the classroom. Many students struggle with comprehension and processing skills when working on word problems. Literature has shown that teaching strategies such as having students justify their answers using solutions in words, drawings or visual tools (Braddock-Hunt, 2015) and encouraging the inclination to check whether their answers make sense (Nosegbe-Okoka, 2004) give students advantages in the process of learning and creating word problems. Another important teaching strategy is to pose word problems tailored to individual needs (Barlow, 2010) as each student can have the opportunity to get assistance in word problems where they struggle most. Overall, unique and targeted teaching strategies are a vital part of supporting students' success with word problems.



**Learning Expectations**  
Instructional goals for our sessions were drawn from the Confrey et al. (2012) learning progressions for Grade 2 Operations and Algebraic Thinking.

## Methodology

- Participants:**
- Four students transitioning from Grade 2 to Grade 3: one male and three females.
  - Student pseudonyms: Ryan, Jess, Katie and Lilly.
- Procedure:**
- Each participated in a 30-minute pre- and post-assessment interview.
  - All students participated in every one of the seven 1-hour instructional sessions.



The cycle shown above was carried out each week over the course of the study. We collaboratively coded qualitative classroom data from each lesson using Newman's Error Analysis Framework (White, 2009). After gathering data, we designed lessons to tend to students' specific learning needs. Findings about students' thinking from qualitative data analysis informed the design of each subsequent lesson.

## Sample Interview Tasks

**Key Task 1:**  
Anna had 43 Lego pieces. She gave some to Leticia and now she has 19. How many Lego pieces did Anna give away?

Task Purpose: To check students' knowledge of "Take From-Change Unknown" word problems.

**Key Task 2:**  
Some children attended a birthday party at 3345 East Westcott in Salisbury, Maryland. At 3 o'clock, 14 children went home and then there were 21 children at the party at 3:15. How many children attended the birthday party?

Task Purpose: To check students' ability to distinguish between relevant and irrelevant information within a word problem.

## Empirical Teaching and Learning Trajectory:

### Initial Assessment Results

#### Key Task #1 – (Take from-Change Unknown)

Ryan struggled with a misconception of the equal sign and task-avoidant behavior. Ryan said his final answer was "19" because that was the number following the equal sign in his self-constructed equation, though his equation contained the correct missing subtrahend. Although the student completed the problem, he displayed task-avoidant behavior such as saying, "Ugh, this one, I don't know" and "This is going to be fun but I also wish I could have stayed home." Jess displayed task avoidant behavior such as choosing to skip the problem as a whole due to a breakdown in reading and comprehension. Katie tried to solve the problem but did so incorrectly also due to comprehension issues. On the other hand, Lilly solved the problem correctly.

#### Key Task #2- (Take from-Start Unknown)

Lilly fell for keyword traps and tended to irrelevant information due to comprehension issues. For example, Lilly read the address in the problem as a potential addend. Both Ryan and Jess chose to skip the task after reading it. Katie failed to distinguish between relevant and irrelevant information. For example, Katie read the time, 3:15p.m., within the word problem and deemed it to be mathematically relevant to the problem. Rather than creating an equation to determine how many people were at the party, Katie drew a clock showing 3:15.

### Lesson 1: Introductory Lesson

The first lesson was an introduction to three-act tasks and carefully thinking about word problems. A three-act task is a set of three short videos that give a scenario. Students watched a three-act task, then as a group, created a word problem based upon what they learned and solved. Overall, this lesson gave insight on student personalities and appropriate objectives for the summer. Based on our observations, we decided to focus on having students solve one or two step addition and subtraction problems with numbers less than 100 and mental strategies for fluently adding and subtracting within 20.

#### Lesson 2: Solving Word Problems with Concrete Objects

Lesson 2 served as a foundation for understanding student thinking and problem-solving skills. The students were given open-task problems and charts to help organize their thinking.



Students tended to find all of the information needed to solve the three-act task but failed to know how to put that information to use.

### Lesson 3: Using Resources to Solve Word Problems

For lesson 3, students were shown a three-act task regarding cookies. The students needed to find how many cookies were taken from the package just by seeing pictures of the original package and how many were left.

Students became confused when trying to keep track of all of the numbers involved in the problem. We decided to give students a more structured way to organize their thinking about each task to help them think more strategically.

#### Lesson 4: Introduction of Organizational Aid

During lesson 4, we tested the new organizational aid we created. It was a "Know, Don't Know and Learned" chart designed to help students read mathematical word problems.

$$\begin{array}{r} 16 - 6 = 10 \\ 23 - 7 = 25 \\ 34 - 2 = 22 \end{array}$$

Know	Don't Know	Learned
16 cookies	6 cookies	10 cookies
23 cookies	7 cookies	25 cookies
34 cookies	2 cookies	22 cookies

Students used the information in the word problem to complete the chart then determined what they needed to find out based on what was left empty in the chart.

### Lesson 5: Independently using the Organizational Aid

In lesson 5, we encouraged fully independent work. Students were able to find important features in word problems and fill out their charts accordingly. They also were able to complete open ended word problems.

For example, students were given a word problem without numbers and were told to create their own problem. Students were able to create a problem and solve it using their organizational chart.

#### Lesson 6: Using Money to Solve Word Problems

During the lesson, students were asked to solve word problems without numbers but instead words that represented numbers such as quarter and dime. The children were able to use coins to help them solve their problems correctly.

What we know	Questions we need to answer	What we learned
1 quarter, 1 dime, 1 nickel, 1 penny	How much money does he have?	31
1 quarter, 1 dime, 1 nickel, 1 penny	How much money does he have?	18
1 quarter, 1 dime, 1 nickel, 1 penny	How much money does he have?	21

#### Lesson 7: Creating Word Problems Independently

During the final lesson, students were able to create their own word problems based upon any lesson they have done this year. With the option of creating a word problem how ever they would like, students were able to produce them on their own.

Barry had 975 stars. Steve had 85 more stars. How many stars did Barry have?

### Post Assessment Results

#### Key Task #1 – (Take from-Change Unknown)

During the post assessment there were many unexpected behaviors from the students. For example, Ryan yet again struggled with the misconception of the equal sign although he got the answer technically correct. Katie showed task avoidant behavior throughout that seems to come from reading and comprehension issues. Jess and Lilly both got the problem wrong but seemed to also struggle with comprehension.

#### Key Task #2- (Take from-Start Unknown)

Lilly re-read the problem multiple times and did not fall for the "keyword traps" yet again. Ryan also was able to use his mathematical skills to solve the problem correctly. Jess completed the problem during this assessment. Katie had task avoidant behavior during this question but ultimately was able to complete the problem with the correct answer.

$$43 - 19 = 24$$

$$14 + 21 = 35$$

**Reflection and discussion:** When engaging with word problems, there are two critical areas of focus: reading/language arts and mathematics. During our research, we found that the six components of Newman's error analysis framework are intertwined and have a ripple effect. When students have a breakdown of understanding in any one or multiple stages in the solving process, their potential for mathematical success is severely impeded. Although our students faced such challenges throughout the study, they did make progress during our instructional sequence. Specifically, the reading strategies we created helped them avoid keyword traps that often interfere with solving word problems. We suggest that other teachers try our reading strategies with their own students when helping them solve word problems.

