The Common Core State Standards writing team (2011) outlined how a student’s learning progresses when introduced to the concept of multiplication. There are agreements from experts on the types of problem situations in multiplication, in grade 3 the focus is on two equal-sized groups of objects and the total number of objects. The team further noted that multiplication problem representations and solutions can be categorized into three levels (pp. 25–26):

- **Level 1**: Students are able to use skip counting to solve their task.
- **Level 2**: Students are using higher level multiplication properties to create and break down problems.
- **Level 3**: Students apply the commutative property to solve their multiplication tasks.

The five streets of mathematical progress, as outlined by Streefkerk and Friel (2001), refer to what is needed for a learner to develop mathematical proficiency. The authors indicate that recognizing all five streets are an important factor to consider. These streets are left to the left from the least thing to the most thing. The authors applied: (1) Fostering Learning Environments; (2) Teaching with the Curriculum; (3) Fostering Learning Environments; (4) Teaching with the Curriculum; and (5) Using the Curriculum.

The five streets are underpinned by a variety of educational articles specific to multiplication learning and instruction. The article Teaching for Mastery in Multiplication by Streefkerk and Friel (2001) describes meaningful teaching and learning methods of the multiplication. The authors present important aspects as to why teaching with meaningful ideas and concepts helps build connections between concepts that are beneficial both for understanding the concepts and performing well on standardized tests. The authors suggest that teachers should explicitly work on these methods that are often not used in the classroom. One of the methods employed by the authors is repeated addition, which uses groups of items, expressing multiplication as the total amount of the items in groups.

The article Direct Modeling and Inverted Procedures: Building on Students’ Informal Multiplication Knowledge by Smith and事业 (1984) proposes the use of the number-totally methods that students use to solve mathematical problems. Direct modeling of the problem situation provides a means through which students can begin to understand multiplication. Direct modelers use physical objects to act out story problems and to reach an answer. Invented strategies can ultimately replace direct modeling. These students are inventing algorithms that reveal how students are making sense of a given problem.

The students who were interviewed created invented algorithms. These students used their excellent recall of multiplication facts, habits of moving through each part of the conversation was audible. Through the transcribed video recording and creating a written transcript of the conversation during the lesson. We also noted all the different emotions and movements throughout the conversation. The students acknowledged that skip counting would be a faster way to count. The students demonstrated the use of the commutative property to solve problems in a variety of ways.

Empirical Teaching and Learning Trajectory:

**Initial Assessment Results**

- **Student Name:** Jake
- **Grade:** 3rd
- **Habitual Inclination:** Counting
- **Conceptual Understanding:** Level Two
- **Procedural Fluency:** Level Two
- **Adaptive Reasoning:** Level Two
- **Evidence:**
  - Jake showed strength in a Conceptual Understanding. He was consistently able to recognize that situations could be represented with multiplication and used facts that he knew to recognize that situations could be represented with multiplication. He was able to transition from concrete to representational language when appropriate. Student also demonstrated better-than-expected reasoning in procedural fluency and adaptive reasoning.
  - Jake was able to use the commutative property to solve a problem that talked about 15 colored pencils in a box. He was able to recognize that it could be solved by using the commutative property. He showed growth in his initial interview where he was not sure if he could solve a problem that talked about 24 colored pencils in a box. He was able to recognize that it could be solved by using the commutative property.

**Instructional Cluster 1**

- **Problem:**
  - The first lesson took place in the classroom with 10 rows of snails. Four rows were shaded blue but shaded with a different color. Each student was given a card with a question and was asked to solve it. The students were given a card with a question and were asked to solve it. One of the questions was: “How many colored pencils do we have in all?”
  - **Initial Strategy:**
    - **Jake:**
      - “I counted each one by 5s. I counted them by each one and made it 20s and 15s and 30s and 10s. I counted them by 5s and 10s and 30s and 10s.”
    - **Mentor:**
      - “You are going every 5 up.”

**Instructional Cluster 2**

- **Problem:**
  - The second lesson took place in the same classroom. The students were given a card with a question and were asked to solve it. One of the questions was: “How many colored pencils do we have in all?”
  - **Initial Strategy:**
    - **Jake:**
      - “I counted each one by 5s. I counted them by each one and made it 20s and 15s and 30s and 10s. I counted them by 5s and 10s and 30s and 10s.”

**Instructional Cluster 3**

- **Problem:**
  - The third lesson took place in the same classroom. The students were given a card with a question and were asked to solve it. One of the questions was: “How many colored pencils do we have in all?”
  - **Initial Strategy:**
    - **Jake:**
      - “I counted each one by 5s. I counted them by each one and made it 20s and 15s and 30s and 10s. I counted them by 5s and 10s and 30s and 10s.”

**Post-Assessment Results**

- **Student Name:** Jake
- **Level:** 2
- **Conceptual Understanding:** Level Two
- **Procedural Fluency:** Level Two
- **Adaptive Reasoning:** Level Two
- **Evidence:**
  - Jake showed some awareness in his thinking through the struggle in approaching a problem that talked about 30 colored pencils in a box. He was able to recognize that it could be solved by using the commutative property. He showed growth in his initial interview where he was not sure if he could solve a problem that talked about 30 colored pencils in a box. He was able to recognize that it could be solved by using the commutative property.

**Reflection and discussion:**

Through this research we have developed tasks that are aligned with the Common Core Standards for 3rd grade multiplication. After having reflected on our work with the students we have found several of the standards more challenging to meet than others. One of the standards was 3.OA.A.4, specifically when students are given the product and have to find the unknown factor. This was challenging to have the student group because they were not familiar with problems that gave the product or start and took on the appearance of more of a division problem. Another standard that was difficult to fully meet was 3.OA.B.5, we worked poorly on the commutative property for this standard. We dealt with the commutative property in multiple lessons but had to gauge whether the students were ready to do this. It was helpful to use visual and manipulatives even when students were at higher levels of the learning progressions. Creating realistic and relevant problems for the students to interact with also helped them with the process of reflecting upon more meaningful mathematical tasks that require deep thinking.