Helping Third Graders Interpret and Make Sense of Word Problems

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Introduction

Students often struggle to engage with word problems (Hunt, 2014). Schools tend to focus on routine word problems that do not require students to put their solutions in context (Nosegbe-Okoka, 2004). As a result, many students struggle when given problems that do not fit within the narrow range of exercises they are ordinarily asked to solve (Christou & Philippou, 1998).

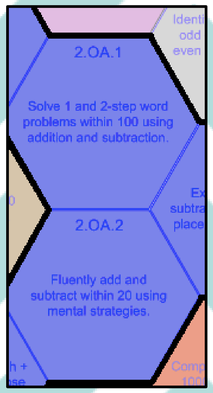
The purpose of our study was to investigate how a group of students entering third grade made sense of multiple types of word problems (Common Core State Standards Initiative, 2010) and to design an instructional sequence to help develop an array of problem-solving strategies. The research questions guiding our study were: Which strategies do students use to solve word problems before, during, and after the instructional sequence? To what extent, and how, do the students' strategies change and develop over the course of instruction?

Literature-Based Teaching Strategies

Previous literature suggests several steps that can be taken to help students with word problems. Eliciting students' interpretations of problems is a foundational formative assessment technique (Dominguez, 2016). Manipulative representations (Murata & Stewart,2017) and diagrams (Ng & Lee, 2009) can be introduced to establish meaningful spaces for student thinking. Throughout the process, making sense of the problem in context should be encouraged rather than using rote key-word approaches (Nosegbe-Okoka,2004).

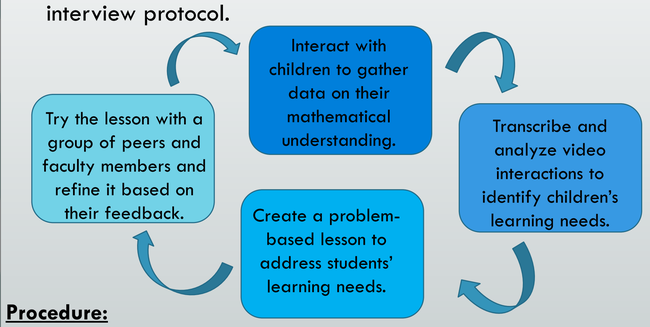
Curriculum Framework:

Instructional goals for our sessions were drawn from the Confrey et al. (2012) learning progressions for Grade 2 Operations and Algebraic Thinking.



Methodology –

- Participants: Four students- 3 females and 1 male
  - Student pseudonyms: Justin, Jessa, Jill, and Joy
- Transitioning from Grade 2 to Grade 3
- 30-minute individual pre- and post-interviews used the same interview protocol.



Procedure:

We video recorded and transcribed each interview and lesson, and retained students' written work. We coded the data collaboratively to qualitatively identify student reasoning patterns. We then made conjectures about tasks that would build our students' abilities to make sense of word problems and designed lessons around them. Before implementing each lesson, we tested it with a group of peers and made revisions. Using this development cycle, we created seven one-hour weekly lessons between pre- and post-interviews. Jessa and Jill both missed one of the weekly sessions.

Key Interview Tasks –

Key Task 1:
15 apples were in a basket. Jose put some more apples in the basket and now there are 26. How many apples did Jose put in the basket?

Key Task 2:
Anna had 43 Lego pieces. She gave some to Alicia and now she has 19. How many Lego pieces did Anna give away?

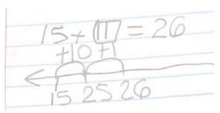
Key Task 3:
Some children attended a birthday party. At 3 o'clock, 14 children went home and then there were 21 children at the party. How many children attended the birthday party?

References

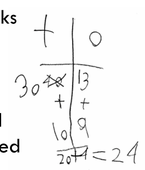
Christou, C., & Philippou, G. (1998). The developmental nature of ability to solve one-step word problems. *Journal for Research in Mathematics Education, 29*(4), 436-442.
Common Core State Standards Initiative. (2010). *Common core state standard for mathematics*. Retrieved from http://www.corestandards.org/math
Confrey, J., Nguyen, K.H., Lee, K., Panorkou, N., Corley, A. K., & Maloney, A.P. (2012). *TurnOnCCMath.net: Learning trajectories for the K-8 Common Core Math Standards*. Retrieved from http://www.turnonccmath.net
Dominguez, H., (2016). Mirrors and windows into student noticing. *Teaching Children Mathematics, 22*(6), 359-365.
Hunt, J.B. (2014-2015). Problems without numbers. *Teaching Children Mathematics, 21*(5), 320.
Murata, A., & Stewart, C. (2017). Facilitating mathematical practices through visual representations. *Teaching Children Mathematics, 23*(7), 405-412.
Ng, S. F, & Lee, K. (2009). The model method: Singapore children's tool for representing and solving algebraic word problems. *Journal for Research in Mathematics Education, 40*(3), 282-313.
Nosegbe-Okoka, C. (2004). A sense-making approach to word problems. *Mathematics Teaching in the Middle School, 10*(1), 41-45.

Initial Assessment Results

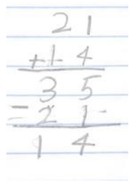
Jessa used multiple strategies; in Key Task 1, for example, she used a number sentence, a hundreds chart, and then an open number line.



Justin created a t-chart for all tasks with double digit numbers to organize tens and ones, as shown in his work for Key Task 2. Justin decomposed 43 as 40 and 3 and 19 as 10 and 9. He then subtracted and regrouped properly.



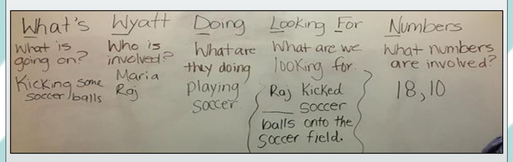
Joy used the keyword strategy in Key Task 3, as well as in other tasks, of "how many" and added the two numbers. We believe that she subtracted 21 because the problem stated that children went home.



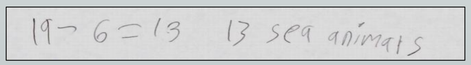
Jill in general, used the counting on strategy to solve word problems. For Key Task 1, she stated: "I counted. I started at 15." She then counted on to 26.

Empirical Teaching and Learning Trajectory:

Lesson 1 Modeling Strategies for Word Problems:
This was an introductory lesson into making sense of word problems to model what to look for in word problems through an acronym that helped them organize their thinking. Students responded well to the "looking for" question in the acronym which aided them in connecting their answer back to the context.



Lesson 2 Making Sense of Word Problems with Clues:
During this lesson, students explained their thought processes for solving one-step word problems involving addition and subtraction. Joy, Jill and Jessa all showed conceptual understanding of place value. Jill showed a new type of thinking by setting up a number sentence and treating it like an algebraic equation to solve a problem.

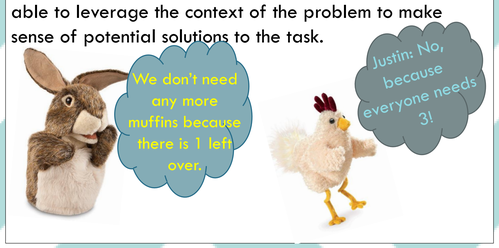


Lesson 3 Solving Open Tasks and Acting Out Problems:
For this lesson, we put students' names into word problems to help them think about acting out problems and contextualizing their answers. For example, one problem was, "Joy has __ gummy bears to eat during the movie. Jill has __ gummy bears to eat. How many more gummy bears does Joy have?" Throughout this lesson, Justin related his answers back to the context of the word problem. Jill, Jessa and Joy all had difficulties connecting the context with the mathematical concepts.

Lesson 4 Word Problems with Extraneous Numbers:
Students were given a word problem containing some numbers that were not needed to solve it. Jill, Justin, Joy and Jessa struggled to connect their operations with the context. When asking Jill how she might solve "how many more girls came at 5 o'clock than at 3 o'clock" she stated "I would add 3" with no explanation as to why she chose her answer.

> Mary's birthday was on the 4th of July. She turned 9. Her party was at 17 Dusty Way. At 3 o'clock there were 10 boys and some girls swimming in the pool. At 5 o'clock, some more girls came to swim. Now there are 20 girls swimming in the pool. How many more girls came at 5 o'clock than at 3 o'clock?

Lesson 5 Solving a Word Problem with Two Answers:
For this lesson, students were given a word problem with two correct answers. After students offered their own solutions, they had to decide if the solution presented by a puppet was correct or not. Joy, Jill and Justin were all able to leverage the context of the problem to make sense of potential solutions to the task.

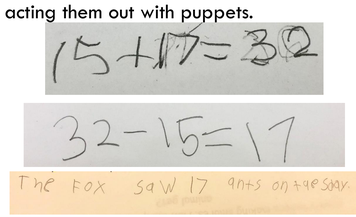


Lesson 6 Word Problems by Using Play:
Students first did a "Notice and Wonder" activity where they built their own word problems based on looking at fruit and vegetables in a basket.

Justin: Uh… I had BLANK fruits and vegetables. I got BLANK more. How many do I have now?

Then, they watched the teachers perform a short skit about Farmer John and his farm animals. Students acted out two word problems and had to determine which of the two skits they acted out were similar to the first skit acted out by the teachers.

Lesson 7:
As in lesson 6, the students first created a word problem using their "Notice and Wonder" where they all had difficulties connecting an operation to the word problem they created as a class. They then worked through three word problems by acting them out with puppets.



Post-Assessment Results

At the end of the study, each student showed significant progress as seen in the post interviews.

Joy and Jessa continued to rely on number sentences and hundreds charts, as they did in the pre-interviews, to solve problems in the post- interview. However, they were able to pull details from the context of the problem to help them justify their operations and verbalize the reasoning behind their answers. Both students showed more consistency in matching their operations with the context of the word problems.

Key Task 1

ZS: Alright, so tell me what you did here?
Joy: So, we started with 15 and we're trying to get to 26.
ZS: Ok so how are we going to solve it?
Joy: I'm going to add something to get to 26.

Jill continued to use the counting on strategy when solving most of the post-interview tasks. Unlike in the pre-interview, Jill was able to communicate contextual details from the word problem and use them to support her mathematical operations when solving both Key Task 1 and Key Task 2 in the post-interview.

Key Task 2

TF: So what are you going to do now?
Jill: I'm taking away some until I get 19.
TF: Okay! Why are you taking ways some?
Jill: Because it says Anna had 43 Lego pieces. She gave some to Leticia. So, that means that Leticia will now have some.

In the pre-interview Justin struggled to relate his answer back to the context of the word problem. However, in the post-interview Justin was able to write answer sentences that connected his answer to the word problem. In the post-interview, Justin also strayed away from decomposing numbers using a t-chart, to relying on hundreds chart and invented strategies, such as counting by tens and fives instead of ones. Justin showed more consistency in his operations and contextualizing his answer back to the word problem.

Reflection and Discussion: After analyzing and collecting data, it was apparent to us that meeting the Confrey et al. (2012) learning progressions for Grade 2 Operations and Algebraic Thinking can be very challenging. The most difficult portions of these learning progressions involved comprehending and making sense of word problems. Initially, students relied heavily on keyword strategies, selecting random numbers in problems, and choosing operations that did not match the problem context. Some students struggled to successfully execute standard algorithms because they were working at a more concrete level using manipulatives. Students had difficulties with non-routine word problems and at times attended only to surface-level features of them. We recommend having students act out word problems because doing so helped our group start to develop a deeper understanding of what problems were asking and aided them in making sense of their solutions within the context of the problems. We also found that having students explain their strategies verbally, use base-ten models, and write their own equations furthered their conceptual understanding of word problems.