Exploring the Roles of Variation and Expectation in Children's Learning of Probability

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Introduction

Probability is a challenging subject to learn, with numerous misconceptions well-documented in previous literature (Jones, Langrall, & Mooney, 2007). It is particularly difficult for students to develop sound intuitions about random behavior (Degner, 2015). However, playing probability games can help build knowledge of probability and random behavior when students systematically conduct trials, record the outcomes, and reflect on the results (Nisbet & Williams, 2009). Such activities have the potential to help students understand the relationship between variation and expectation, which is at the heart of learning probability (Watson & English, 2016).

Purpose

The purpose of the study was to investigate students' thinking about variation and expectation in probability and to design a game-based instructional sequence to help their thinking develop.

Research Question

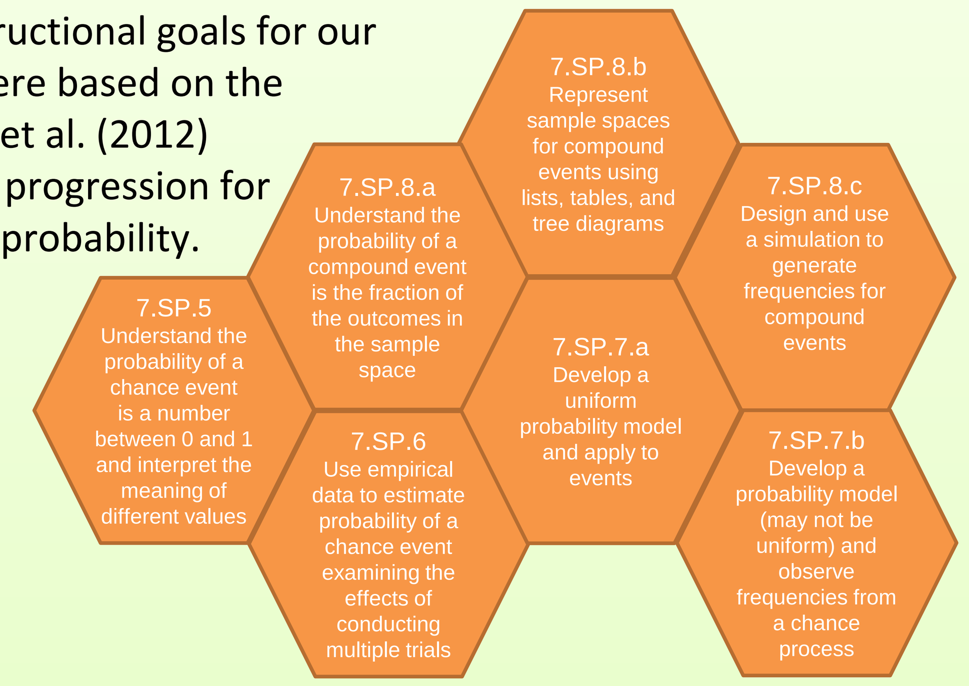
How do students think about variation and expectation before, during, and after a game-based instructional sequence designed to help them build probabilistic knowledge?

References

Degner, K. (2015). Flipping out: Calculating probability with a coin game. *Mathematics Teaching In The Middle School, 21*(4), 244-245.
Jones, G.A., Langrall, C.W., & Mooney, E.S. (2007). Research in probability: Responding to classroom realities. In F.K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 909-955). Charlotte, NC: National Council of Teachers of Mathematics and Information Age Publishing.
Nisbet, S., & Williams, A. (2009). Improving students' attitudes to chance with games and activities. *Australian Mathematics Teacher, 65*(3), 25-37.
Watson, J.M., & English, L.D. (2016). Development of probabilistic understanding in fourth grade. *Journal for Research in Mathematics Education, 47*(1), 28-62.

Curricular Framework

The instructional goals for our study were based on the Confrey et al. (2012) learning progression for Grade 7 probability.



Literature-Based Teaching Strategies

In our research, we drew upon teaching strategies documented in existing literature to help students develop more robust probabilistic reasoning. Nisbet and Williams (2009) expressed how creating a positive learning environment can help improve students' attitudes towards mathematics. Using games in the lessons can provide a social aspect to the classroom that also leads to an increase in interest overall. McCoy, Buckner, and Munley (2007) expressed that using games, especially multicultural games, provides a rich and interesting context for applying important probability concepts. Truran (2016) outlined the importance of probability in education, focusing on experimental and theoretical probability and using a "guess-check-revise-check" method when conducting simulations.

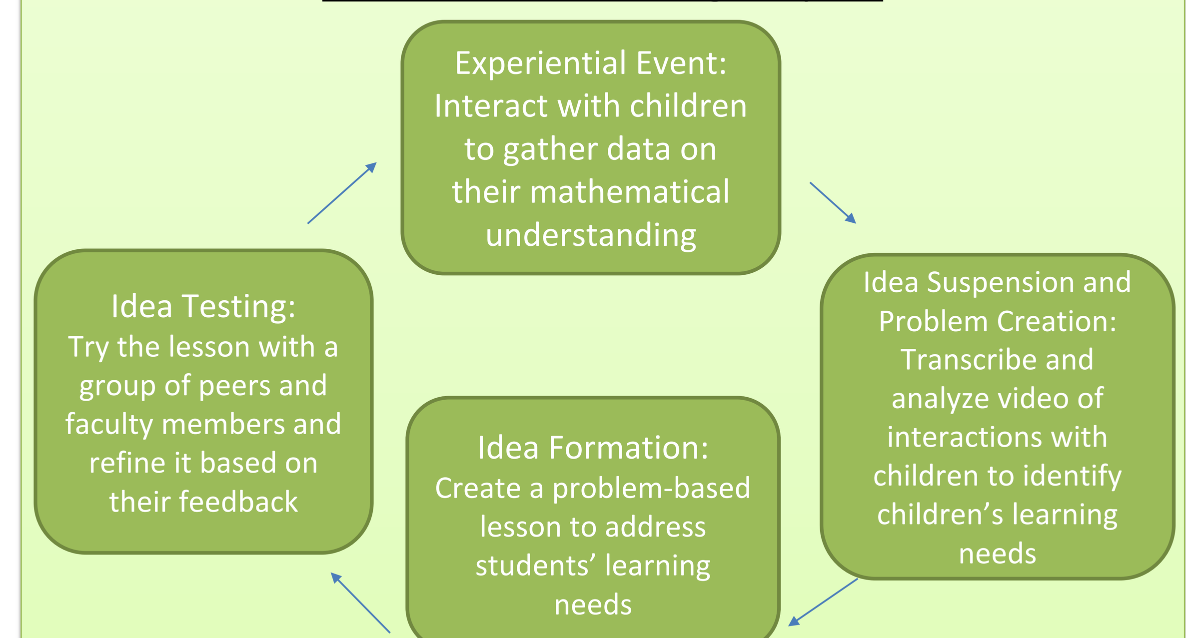
References

Confrey, J., Nguyen, K. H., Lee, K., Panorkou, N., Corley, A. K., & Maloney, A. P. (2012). *TurnOnCCMath.net: Learning trajectories for the K-8 Common Core Math Standards*. Retrieved from http://www.turnonccmath.net
McCoy, L., Buckner, S., & Munley, J. (2007). Probability games from diverse cultures. *Mathematics Teaching in the Middle School, 12*(7), 394-402.
Nisbet, S., & Williams, A. (2009). Improving students' attitudes to chance with games and activities. *Australian Mathematics Teacher, 65*(3), 25-37.
Truran, J. (2016). What is the probability of...?. *Australian Mathematics Teacher, 72*(3), 59-60.

Methodology – Participants and procedure

Our group consisted of four students, two male and two female, who had all completed sixth grade and were advancing into the seventh grade. We assigned the students the pseudonyms Buddy, Kari, Violet, and Robert. They participated in a 30 minute pre-interview and a 30 minute post-interview as well as 7 weekly one-hour instructional sessions. (Robert missed two of the weekly sessions and the post-interview). The pre-interview and post-interview scripts were identical and contained items from the National Assessment of Educational Progress (NAEP).

Instructional Design Cycle



We repeated the instructional design cycle above each week to understand and analyze the students' reasoning and refined each subsequent lesson based on the analysis (Ricks, 2011). Our process started with the experiential event, via our pre-interviews with the students. From there, we analyzed the interview, formed a lesson based on their current understanding, and refined the lesson with our peers and faculty members.

References

Ricks, T.E. (2011). Process reflection during Japanese lesson study experiences by prospective secondary mathematics teachers. *Journal of Mathematics Teacher Education, 14*(4), 251-267.

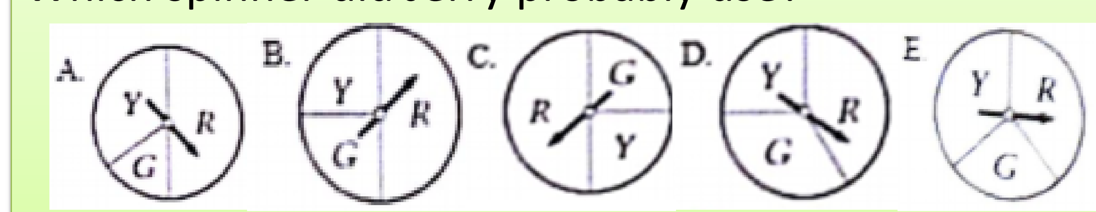
Methodology – Key NAEP Interview Items

Key Item #1:

Jerry spun one of the spinners below 1,000 times and obtained the results shown in the table above. Which spinner did Jerry probably use?

Results

G	157
Y	352
R	491



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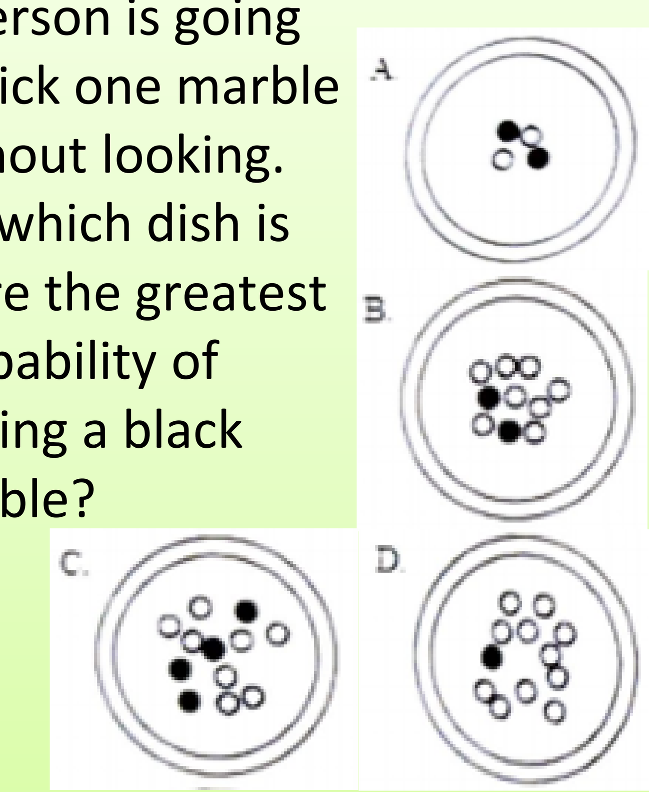
Key Item #2:

A person is going to pick one marble without looking. For which dish is there the greatest probability of picking a black marble?



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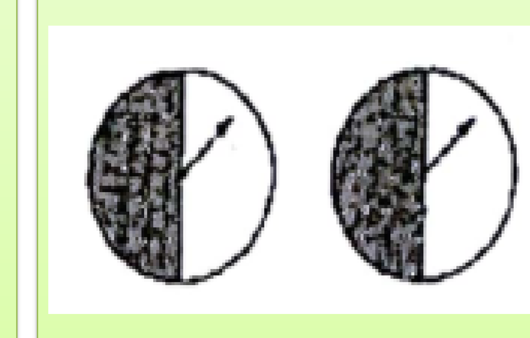
Key Item #3

The two fair spinners shown below are part of a carnival game. A player wins a prize only when both arrows land on black after each spinner has been spun once. James thinks he has a 50-50 chance of winning. Do you agree?



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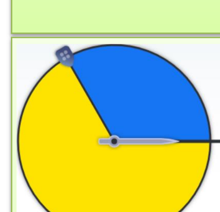
Each interview and lesson was video recorded and transcribed. Transcripts were coded qualitatively and inductively in collaboration with faculty mentors. While coding, we focused on identifying normative and non-normative student reasoning patterns. We also carried out axial coding to continuously group and re-group codes to produce a comprehensive portrait of the reasoning students exhibited.

Initial Assessment Results

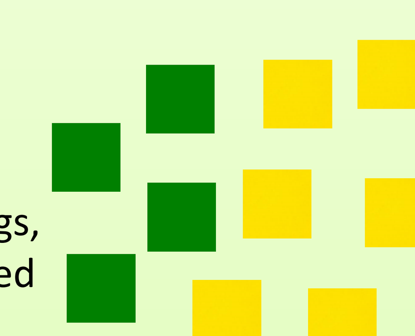
Comparing theoretical and experimental probabilities posed a challenge for Violet. She thought yellow and green would occupy the same amount of space on a spinner (Key Item 1), even though yellow appeared in the results far more often.
One student, Buddy, used odds ratio-like expressions for each problem, as when quantifying probabilities in Key Item 2 (1:1, 1:4, 1:8, e.g.).
Three of the four students had trouble with compound events. They believed there was a 50-50 chance of winning the carnival game in Key Item 3 when there was actually only a 25% chance. Robert's interview response is illustrative:
"There's an equal amount on each side and when he spins he could have the same amount of getting it [white or black]."
All of the students seemed to have a basic understanding of what probability/chance was, giving us an opening for our first lessons.

Empirical Teaching and Learning Trajectory:

Lesson 1: Determine which coin outcomes (heads versus tails for part one, HH & TT versus HT for part two) have a better chance of occurring by flipping coins and recording the results.	Student responses were generally brief but in occasionally showed possible beginnings of combinatorial reasoning. Kari: "If you flip it, it's still the same, but like are still two independent coins."
Lesson 2: The task was to have students examine simple events by creating bi-colored spinners for different probabilities ranging from impossible to certain.	Buddy and Kari were able to order the spinners using probability vocabulary. Violet understood concepts of impossible/certain, but struggled when a spinner split into fours was used.
 Lesson 3: Students predicted how many spins would land on each section if the spinner were spun 50, 100, and a 1000 times.	Kari predicted 33 blues in 100 spins: "I did 100 divided by 3, because there's three sections." Both Robert and Kari noticed that conducting more spins leads to more accurate results.

Lesson 4:
Task: Students pulled cubes from "mystery" bags labeled A to E to determine the color compositions of cubes in each bag.
Reasoning: Buddy and Kari showed a distrust for the idea of sampling with replacement. Robert, however, was able to recognize the value of many draws from the different bags.

Lesson 5:
Task: Students worked together to determine the color composition for three "mystery" bags, each of which contained a 4:6 ratio, by drawing samples from each bag. Spinners were also created to match each sample pulled.



Reasoning: Buddy and Kari again saw the importance of the value of many draws and the value of many samples. Violet, however, did not allow variation when they compared the bag size of 20 the corresponding pie chart. She felt that the ratio in the bag had to perfectly match the ratio of the spinner.

Lesson 6:
Task: Students made conjectures about what a hidden spinner looked like after being given the results of some spins on it.
Reasoning: Both Buddy and Kari seemed to allow for variation in their spinner models; both of them were not too surprised if their estimation wasn't exact. When Robert created a spinner based on results close to 50/50, he started with a spinner split down the middle and then adjusted the halfway mark to match the results.

Lesson 7:
Task: Students worked together to create spinners to match bags marked with a number of yellow and green cubes. They also created bags that would fit the idea of "equally likely."
Reasoning:
All four students seemed a bit confused when performing samples on a bag with only one green and one yellow cube, likely due to the small population size. The students were also very keen on making the sample size a large percentage of the total population (25%-50%).

Post-Assessment Results

Compared to the first interview, Violet did show some slight improvements. However, contextual details still confused her in many of the questions. Both Kari and Buddy showed much more improvement, using strong proportional reasoning in many of their answers.
In many of the problems, Kari used fractions to relate areas of spinners to expected values. For example, one problem had a spinner split up into three parts, with a 2:1:1 ratio. She recognized that the smaller portions were quarters and was able to correctly estimate how many would land in a quarter portion out of 300.
Compound events still posed trouble for the students, something we did not have much time to address. Only Buddy was successfully able to state that the carnival game had a 25% chance to win (key item #3). He listed out the four different outcomes and determined how many would result in a win. Unfortunately, Robert did not attend the final post-assessment due to responsibilities elsewhere.

Reflection and discussion:

Many challenges surfaced as we designed and taught lessons. Students had difficulty understanding sampling with replacement; they tended to not trust this sampling method. Students also generally wanted the sample size to be a large percentage of population and would be worried about pulling the same cube twice in a single sample. The seventh grade learning progression guiding our study involved compound events. Our experience suggests that several more lessons would be needed to prepare students to study this topic; moving from a basic understanding of probability to understanding compound events is not a trivial matter. Although probability is not addressed until Grade 7 in the Common Core, it would be helpful for teachers in earlier grade levels to develop children's understanding of beginning probability concepts so that the seventh grade curriculum is not overwhelming. Our study provides examples of how such understanding can be developed through games and other concrete activities.