Building Conceptual Understanding of Multiplication

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Introduction -

Multiplication and division are foundational to students' learning of a wide array of mathematical ideas (National Council of Teachers of Mathematics, 2008). Children should develop conceptual understanding and procedural fluency with multiplication and division in Grades 3-5 (Common Core State Standards Initiative, 2010). However, in these grade levels, children are often taught multiplication and division in a strictly procedural manner without emphasis on conceptual understanding (van de Walle, Karp, & Bay-Williams, 2013).

With this in mind, we sought to design an instructional sequence to help a group of children entering fourth grade develop both conceptual understanding and procedural fluency for multiplication and division.

The research question guiding our study was: What is the nature of the children's conceptual understanding and procedural fluency for multiplication and division before, during, and after the instructional sequence?

References:
Common Core State Standards Initiative. (2010). *Common core state standards for mathematics*. Retrieved from http://www.corestandards.org/math
National Council of Teachers of Mathematics. (2008). *Curriculum focal points for Prekindergarten through Grade 8 mathematics: A quest for coherence*. Reston, VA: NCTM
van de Walle, J.A., Karp, K.S., & Bay-Williams, J.M. (2013). *Elementary and middle school mathematics: Teaching developmentally* (8th ed.). New York: Pearson.

Literature Review -

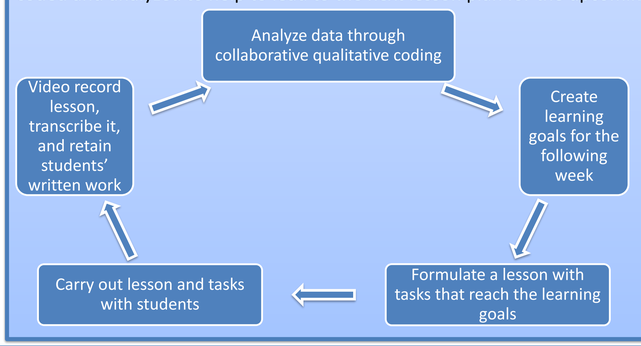
The following Common Core State Standards were used to set learning goals for our instructional sequence (Confrey et al., 2012):
3.OA.1-Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.
3.OA.3- Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
4.OA.1-Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

Teaching Strategies from the Literature:
In order to successfully develop students' understanding of multiplication in the deepest level, teachers must go beyond procedural instruction. In the lessons for this study, we gave the students the freedom to actively create their own multiplication stories that engaged their interests (Sullivan & McDuffie, 2009). We encouraged the students to talk through their reasoning with peers in order to grasp a deeper understanding of their work (Whitin & Whitin, 2008). The students worked with multiple mathematical representations in order to learn various methods of multiplication and how they may be used and formulated; such as equal grouping/repeated addition, arrays, and number lines (Anghileri, 1989).

Methodology -

Participants- There were 2 girls and 2 boys who had just completed the third grade and were entering fourth.
Student Pseudonyms- Kasey, Isaac, Ryan, Chloe.
Student Participation Rate- 100% for Isaac and Ryan, Chloe missed 1 session and Kasey missed 2.
Duration- We taught seven weekly 1-hour sessions. Before the first lesson each child participated in a 30-minute individual interview. A 30-minute post-interview using the same task script was administered after the last lesson.

Procedure
Each week the previous lesson was used to determine tasks for the following lesson. The students' written work was collected along with video and audio from the lesson that was transcribed weekly. The transcripts were then coded and analyzed to help to lead to the next lesson plan for the upcoming



Methodology- Data gathering and analysis

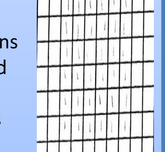
Pre and Post Interview: Key Tasks

Key Task 1: Ten rows of snails. Four snails in each row. How many snails?

Key Task 2: For a school field trip 72 students will be traveling in 9 vans. Each can will hold an equal number of students. The equation shows a way to determine the number of students that will be in each van.
72 ÷ 9 = ?
This equation can be rewritten using a different operation. Place the operation and number pieces we will provide you in the proper boxes

___ ___ ___ = 72

Key Task 3: The grid shows how much wall space the art teacher can use. Use the grid to create a rectangular array showing how the art teacher might arrange the tiles on the wall. Select the boxes to shade them. Each tile should be shown by one shaded box.

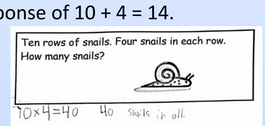


References:
Anghileri, J. (1989). An investigation of young children's understanding of multiplication. *Educational Studies in Mathematics*, *20* (4), 367-385.
Confrey, J., Nguyen, K. H., Lee, K., Panorkou, N., Corley, A. K., & Maloney, A. P. (2012). *TurnOnCCMath.net: Learning trajectories for the K-8 Common Core Math Standards*. Retrieved from http://www.turnonccmath.net
Sullivan, D.A., & McDuffie, A. R. (2009). Connecting multiplication to contexts and language. *Teaching Children Mathematics*, *15* (8), 502-510, 512
Whitin, P., & Whitin, D. (2008). Learning to solve problems in primary grades. *Teaching Children Mathematics*, *14*(7), 426-432.

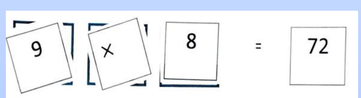
Empirical Teaching and Learning Trajectory:

Initial Assessment Results

Key Task 1: In this task, the students were to solve the multiplication problem using whichever method worked for them. Isaac, Ryan, and Chloe all created the equation 10 x 4 = 40. Ryan's work is shown below. Kasey gave the response of 10 + 4 = 14.



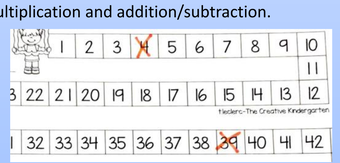
Key Task 2: In this task, the students chose from a pile of numbers and operations to fill in the blanks. Isaac, Ryan, and Chloe all inserted 9 x 8 while Kasey did not attempt the problem.



Key Task 3: Only Ryan and Chloe attempted this task. It took Ryan two attempts to correctly complete the problem. Chloe was not able to solve the task. She attempted the task multiple times but could not find factors of the number of tiles to correctly create a rectangular array.

Lessons 1 - 2: Connecting Skip Counting to Multiplication

Lesson One: Our first lesson was designed to continue developing the students' ability to skip count with different numbers in order to prepare them for multiplication. We constructed a game that incorporated skip counting with die and using a number line to demonstrate "hops" with skip counting to help the students to connect skip counting and multiplication as equal -sized groups. Skip counting by threes was the greatest struggle. Some students used a 100s chart to count by 1s.
Lesson Two: Lesson 2 was designed to further develop students' abilities to skip count and to introduce multiplication number sentences. We redesigned the skip count game using two dice, instead of one, so that this time each student was able to form a unique multiplication number sentence each roll. This gave students the ability to practice with different numbers almost every turn. They also solved the multiplication problems their peers obtained to remain actively engaged. Students struggled with their fact fluency. The students also created their own number sentences using multiplication and addition/subtraction.



Kasey's work

Transitional Lesson 3

The students worked on multiplication word problems.
Ex word problem: "The Ferris wheel costs 5 tickets to ride. How many tickets does it cost for 5 people to ride?"
Students also completed exercises to assess their fact fluency. The students were at very different levels. Kasey only managed to solve the 1s and 0s facts. Isaac and Chloe had similar abilities and were able to solve the majority of problems with the exception of some 6s, 7s, and 8s facts. Ryan solved all problems with very few issues and only took a bit longer with the 8s facts.

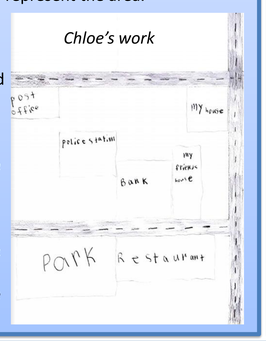
Lessons 4 - 5: Grouping and Arrays

Lesson Four: The students were introduced to the smart board and were asked to create rectangular arrays from multiplication sentences. They then explained their grouping methods to one another and the commutative property was briefly discussed. A demonstration was done to show the students how to find factors of a number in order to create a rectangular array. The students used this knowledge to build rectangular arrays and determine their areas. All students were able to build and determine the areas of the arrays. Most of the students used their fact fluency in order to solve for the area but occasionally counted the inner blocks of the shape by ones to solve for the area; Kasey did this the entirety of the lesson.

During the smartboard array activity the difficulties students had with making connections between building arrays and solving a multiplication sentence with repeated addition became apparent. Chloe struggled with a 4 x 9 array, crafting the addition sentence 4 + 9 + 4 + 9 + 4 + 9 + 4 + 9= 36 to represent the area.

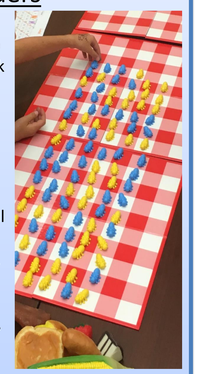
Lesson Five: The students constructed pools using rectangular arrays and calculated areas of figures together on an interactive board. They then were given grid paper to construct their own towns made of pre-determined rectangular arrays (e.g., 24 blocks or 3 by 5). This allowed them to learn the different ways that arrays can be built with the same area and gave them the opportunity to be creative and share their work.

Chloe's work



Lessons 6 - 7: 100 Hungry ants and remainders

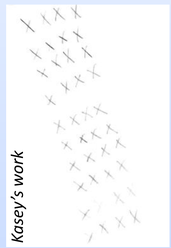
Lesson 6: The students were handed back their town maps to finish. They then hung their work up for a gallery walk where they explained their favorite building by telling their peers the building's area and dimensions. Next, each student was given 24 ants and red checkered picnic boards. The students explored all rectangular arrays that they could make with the number 24. The students got an extra ant (making 25 ants) to explore the topic of division and remainders, attempting to create arrays for equal rows of 2, 3, 4, and 6.



Lesson 7: The students were collectively given 100 ants to manage together, working as a team. The students were instructed to find all of the ways that 100 ants could be placed into a rectangular array. The students worked together with Ryan leading the group with his strong understanding of factors. The students then explored division again with rectangular arrays and remainders using numbers that do not go into 100 evenly such as 3, 6, and 7. To finish the lesson, the students played the skip counting game with the two dice. Very strong fact fluency was shown during this portion of the lesson and the students were able to skip count to move their pawns on the board more efficiently and quickly.

Post - Assessment Results

Key Task 1: All four students were able to correctly answer this problem. Isaac, Ryan, and Chloe knew that 4 x 10 = 40. Kasey used an alternative method for solving the multiplication problem by drawing her own rectangular array.



Kasey's work

Key Task 2: In this task Isaac, Ryan, and Chloe all reasoned as they did in the Pre-interview. Kasey chose the numbers 9 and 8 for this task but used addition as the operation instead of multiplication.

Key Task 3: Ryan was able to solve the task as he did in the Pre-interview. Chloe was now able to complete the task using her knowledge of rows and columns in rectangular arrays. Isaac and Kasey counted the number of tiles incorrectly, but the arrays they created were correct for their incorrect values (e.g., a 3x3 created for 9 tiles).

Chloe's work

Reflection and discussion: During our study students demonstrated the abilities to skip count, construct rectangular arrays, and solve multiplication sentences. The main challenge was to have our students connect all of these skills so that they were able not only to solve simple procedural tasks but to have a deep conceptual understanding of multiplication. Developing students' conceptual understanding was a high priority because procedural understanding alone is fragile and insufficient for making sense of complex problems students will encounter in the future. We helped develop students' conceptual understanding by prompting them to think about how skip counting, repeated addition, and areas of rectangular arrays are all connected to multiplication. We recommend that other teachers work to help students make these connections as well, incorporating communication with peers and allowing students to have creative flexibility with tasks, as modeled in our research results.