On a recent National Assessment of Educational Progress (NAEP) test, less than half of fourth grade students performed at or above the "proficient" level (National Center for Education Statistics, 2013). Fractions, in particular, caused difficulties for students. For example:

- 45% of 4th graders were unable to add 3 fractions with like denominators (Fourth Grade NAEP test item, block: 2013-4M3, Number 9)
- 26% of 4th graders were able to correctly add 3 fractions together with unlike denominators with a model (Fourth Grade NAEP test item, block: 2013-4M7, Number 13)

Achievement level results in 4th Grade Mathematics (National Center for Education Statistics)

Purpose: The Common Core State Standards (National Governors Association & Council of Chief State School Officers) aim to ensure that all students develop understanding of fraction equivalence in fourth grade. Fourth-graders are also to add and subtract fractions with like denominators. In our study, we aimed to study how a group of students finishing fourth grade might begin to understand addition and subtraction of fractions with unlike denominators.

Research Question: How can students’ mathematical proficiency be developed for understanding addition and subtraction of fractions with unlike denominators?

Empirical Teaching and Learning Trajectory:

Initial Assessment Results
On the pre-assessment interview, students tended to have difficulty explaining why fractions were equivalent. For example, Steven was given the following problem:

1. Are the fractions from part a & b equal? Explain your answer with a drawing. Steve: There is the same size in the same length. I say that the other fraction is as size as size. I want to have a fraction of the same length. Steve: Yes

Students also had trouble adding and subtracting fractions with unlike denominators. For example, Vanessa was asked to respond to the item below:

15. How do we need to do for this problem? 
What do we need to do for this problem?

15. Fractions from part a & b equal? Explain your answer with a drawing. Steve: There is the same size in the same length. I say that the other fraction is as size as size. I want to have a fraction of the same length. Steve: Yes

Institutional Sessions (Weeks 2.4)
For our first lesson, week 2 of the program, the students were asked to generate an original fraction of ⅓ and asked to make equivalent fractions. The following image depicts Lauren splitting her garden into equivalent fractions.

Institutional Sessions (Weeks 3.5)
For our first lesson, week 3 of the program, the main focus was on finding common denominators with the fractions given by playing a game called one and out. During this time we were trying to address the weakness from week 2.8. We wanted the students to understand fractions as being a single number instead of referring to it as two entities (numerator & denominator).

Institutional Sessions (Weeks 6, 7, 8)
During weeks 6-7, we focused on addition and subtraction of fractions with unlike denominators and also included a few addition problems with fractions that had like denominators.

- During Week 6 students made the connection between the common denominator and size of the pieces which was a continuation of the lesson previously taught during weeks 4 and 5.
- During week 7, students were given an open ended problem. They were a given a scenario that provoked them to come up with their own questions referring to the total amount of pizza left.
- During week 8 students played a game called "Jump." They were asked to roll 2 dice and the total that the die added up to was the problem they were suppose to solve. Each student would solve the addition or subtraction problem as individuals. Below is an example of the Jump Game.

Post-Assessment Results
On the post-assessment interview, students were asked to explain if the answer of 5/12 was reasonable. For example, Steven was given the following problem:

4. 12 12 5
5. 12.

Reflection and discussion: Based on our research during this project we saw that the students had trouble with the CCSSM Standard 5.NF.2: estimate and solve word problems using addition and subtraction of fractions with unlike denominators. Students struggled to understand the meaning behind finding a common denominator; therefore the difficult points in the learning progression were the ones that addressed common denominators. Although they knew that you cannot add fractions straight across when the denominators are different they didn’t know how to actually find a denominator that could be used between fractions. Since students were very familiar with making equivalent fractions and drawing fraction models to represent their understanding this was our gateway into addition and subtraction with unlike denominators. The key helpful tools to move the students beyond creating equivalent fractions and onto finding common denominators were the fraction bars and virtual manipulatives. Both manipulatives helped the students to reinforce how to make equivalent fractions and visualize why finding a common denominator was necessary to perform the operations. Students needed to make the connection that the “pieces” of the whole had to be made the same size in order to add or subtract the fractions. In one lesson to obtain this we provided the students with fraction bars to measure out the sum of their fractions. This was a vital part of our learning progression. We find this standard significant in terms of requiring students to visually represent their ability to find common denominators for a set of fractions so that they are able to grasp this concept entirely.