Developing Students' Conceptual Understanding of Place Value and Decimals

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Introduction

According to Ashlock (2010) and Durkin and Rittle-Johnson (2014), students have difficulties learning and understanding decimals. Common misconceptions include:

1. Students often attempt to apply their knowledge of whole numbers to decimals. For example, students may believe that 0.45 is greater than 0.8 because 45 is greater than 8 (Durkin & Rittle-Johnson, 2014).
2. Students often believe if there is a zero at the end of a decimal number, that the value increases. For example, students might believe that 0.89 has a lesser value than 0.8900 (Durkin & Rittle-Johnson, 2014).
3. Students are often told to "line up the decimal point" when adding and subtracting with decimals, but do not understand that they are actually lining up the place values of each number. For example, they may believe that 0.7 + 0.7 = 0.14 (Ashlock, 2010).

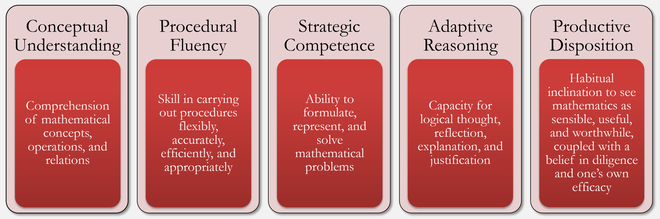
The purpose of this study was to examine students' thinking about whole number place value and the base ten system and how their understandings of these topics influenced their learning of decimals and decimal computations.

Research Question

How can students' proficiency be improved in the areas of whole number place value, decimal place value, and decimal computation?

Theoretical Framework

The Strands of Mathematical Proficiency model (National Research Council, 2001, p. 116) was used to shape our approach to the research.



Procedural fluency is often the most heavily emphasized of the 5 strands. Rote memorization is often the focus of lessons, and students are expected to perform computations without conceptual understandings. In order to guide students toward mathematical proficiency, teachers need to focus on curricula, lesson planning, and the development of activities that focus on student growth in all five of the strands of mathematical proficiency.

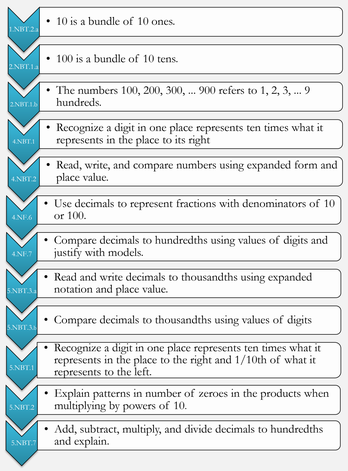
We drew upon mathematics education literature for teaching strategies and principles likely to develop students' mathematical proficiency for decimals. These included:

- -Building upon students' prior knowledge, connecting what they already understood to new information (Carpenter et al., 2015)
- -Building upon students' knowledge of place value to develop their understanding of decimals (Ashlock, 2010)
- -Using visual models to illustrate the conceptual structure of decimals (National Governors Association & Council of Chief State School Officers, 2010)
- -Using inquiry-oriented instruction including tasks from outside-of-school contexts to encourage student engagement (Reys et al., 2014).

Methodology: Participants and Procedure

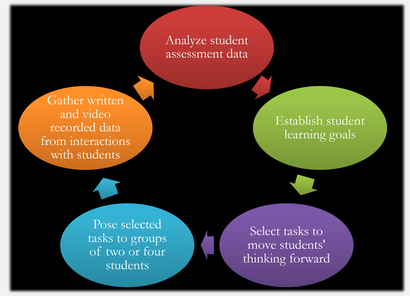
Four students participated in the study, Alex, Bethany, Christina, and Daniel (pseudonyms). These students were entering fifth grade in fall 2015. During the summer, students participated in an initial assessment interview, seven 1-hour tutoring sessions, and a post assessment interview. All four students attended each session and participated actively by answering questions, adding their thoughts and opinions during class discussion, and helping one another solve problems.

Learning Trajectory for Place Value and Decimals



The standards shown to the left are the portions of the trajectory that were used during instruction (Confrey et al., 2012).

PATHWAYS Cycle of Integrated Teaching and Research



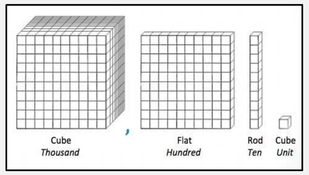
Methodology: Data Gathering and Analysis

At the beginning of this study, each student participated in an initial 30-minute interview that assessed his or her prior knowledge on items related to the learning trajectory for place value and decimals (Confrey et al., 2012).

Each student answered the same set of interview question at the conclusion of the summer. Data was analyzed from the pre and post interviews showing: where the students' learning started, where the students made gains, where the students did not make gains, and where the students made the most gains collectively.

Sample Interview Questions

1. What would be the value of the next block to the left [of the thousands cube]?
2. Add 2.4 + 10.03
3. Subtract 12.0 – 0.145
4. Multiply 0.2 x 3



Each instructional session and interview was video recorded. After each lesson was taught, we discussed how the lesson went, which students seemed to have a good understanding of what was taught, and preliminary next steps for instruction. We then transcribed the videos. After transcribing, we coded the transcripts and student work using the Five Strands of Mathematical Proficiency and made conjectures about tasks and teaching strategies to help advance students' thinking.

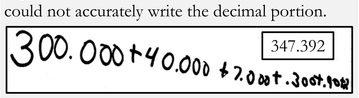
Empirical Teaching and Learning Trajectory:

Initial Assessment Results

All students were able to accurately determine the name and value of the base ten blocks, but none of the students could determine the value of the block immediately to the left of the thousands block. This showed that students were not completely sure of the patterns found within the base ten system.

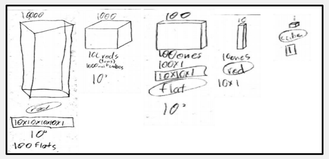
After reviewing the initial assessments of all four students, it was clear that the students were entering the program with a wide range of ability levels. Alex and Bethany demonstrated procedural fluency, by using algorithms to solve the computations. These two students, however, lacked conceptual understanding, as shown by their inability to apply their knowledge to an unfamiliar problem. Alex was able to solve 0.2x3, but could not figure out where to start when trying to solve 2.5x0.5. Christina and Daniel were both unable to read the decimals correctly and could not perform the addition, subtraction, and multiplication of decimals problems.

There were also times when students thought they were able to determine the correct answer, but in fact had the wrong answer. In the initial interview Bethany stated that she was "good" at expanded form, but her work showed that she could not accurately write the decimal portion.

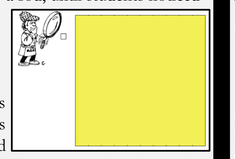


Instructional Cluster 1

During the first lesson, students demonstrated their knowledge of whole number base ten blocks by discussing what they knew about the blocks and discovering patterns that can be found among the blocks. Students created a chart that displayed important information about each block:



Students did not demonstrate *conceptual understanding* that each place value is ten times bigger than the place value to it's right and ten times smaller than the place value to it's left. We addressed this problem by having students figure out how many of each block was in the next block, i.e. how many unit cubes make up a rod, until students noticed the pattern.

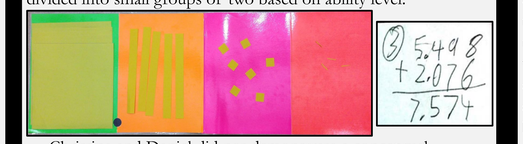


In the second lesson, students were given a square that represented a 2-dimensional unit cube that they cut into tenths, hundredths, and thousandths pieces to represent each place value. This activity demonstrated that each piece had to be ten times smaller in order to accurately represent the place value to its right.

In the third lesson, students were given playing cards with the numbers 1 through 9. Students selected cards and made the smallest number possible, by putting a number in the ones, tenths, hundredths, and thousandths place value, using the numbers they selected. Students *strategic competence* was strengthened by using manipulatives to represent a variety of numbers throughout these lessons.

Instructional Cluster 2

Once students attained a conceptual understanding of place value, students were introduced to adding and subtracting with decimals. Students used manipulatives to represent each number and placed the pieces on a place value mat, which helped students line up the place values when they were adding. The students were divided into small groups of two based on ability level.



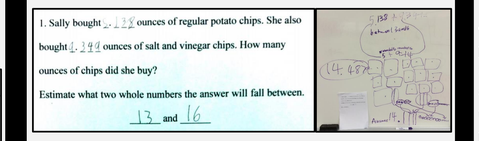
Christina and Daniel did not demonstrate a conceptual understanding of regrouping when adding. In order to help these students, they completed various addition problems that required regrouping. As they regrouped, they were asked to explain why they did so. This helped to develop students' adaptive reasoning. To further their understanding of regrouping with addition, addition problems were included in the subtraction game for the next lesson.

Students were introduced to subtraction of decimals by playing a board game. Each student was given a set of question cards that were specifically designed for his or her ability level. As students played the game, they answered a series of subtraction and addition problems, while using the manipulatives to support their understanding and strategic competence.

Instructional Cluster 3

In the sixth lesson, students were introduced to multiplication of a whole number and a decimal, for example, 3 x 0.8. Christina and Daniel lacked *conceptual understanding* of multiplication that was needed to use the manipulatives to represent the problem. For 3 x 0.8, students would set up 3 groups of 8 tenths pieces. In order to help students gain a better understanding of how to set up the problem, students were given the opportunity to work with a partner and set up problems that were all the same format. Students demonstrated their *adaptive reasoning* skills by explaining how they set up their manipulatives and how they found their answer.

The seventh and final lesson consisted of review of addition, subtraction, and multiplication with decimals. In order to develop students *productive disposition*, they were given word problems that dealt with real world situations.

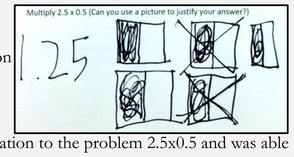


Parallel, open ended tasks were chosen in order to differentiate instruction, but did so in a way that allowed students to have a meaningful discussion about how they solved each problem, enhancing their *adaptive reasoning* skills. Students chose numbers that they were capable of working with. Students then had to estimate what two whole numbers the answer would fall between, and then draw models of the manipulatives to prove that their answer was correct.

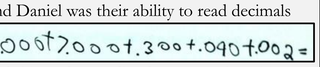
Post-Assessment Results

In the post interview, Alex and Bethany demonstrated improved *conceptual understanding* of the base ten blocks by accurately indicating the value of the block immediately to the left of the thousands is 10,000. However, Christina and Daniel were still unable to accurately state the value of the block.

Alex and Bethany relied on algorithms to solve the addition, subtraction, and multiplication problems given, but were also able to explain their answers using *adaptive reasoning* and drew pictures to demonstrate their *strategic competence.* Alex was applied his understanding of multiplication to the problem 2.5x0.5 and was able to draw a picture to represent the problem.

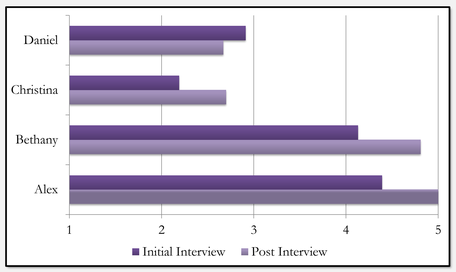


A significant improvement for Christina and Daniel was their ability to read decimals correctly, demonstrating an understanding of decimal place value. Bethany also demonstrated this understanding by accurately writing the number 347.392 in expanded form and as a mixed number.



The initial and post interview data was analyzed by rating student responses on a scale of 1 to 5. A score of a 1 meant that students did not show any understanding of the problem and a score of a 5 meant that students showed a complete understanding of the problem. The results of both interviews were compared to see if students performance Rating increased or decreased. Alex, Bethany, and Christina's overall score increased, while Daniel's score decreased.

Initial and Post Interview Results



Reflection and Discussion

The sequence of the learning progression was easy to follow and each standard transitioned well to the next. The most difficult part of this learning progression is developing a conceptual understanding of each standard. In order to address these difficulties, students should experience conceptual learning, through individual and group work. The results of this study have demonstrated the importance of spending ample time on each standard before moving on to the next one. It is essential that students have multiple experiences within each standard. One shortcoming of this study was moving too quickly through the standards, which did not allow all students to gain a full understanding.

In order for teachers to teach this progression, pre assessment is imperative to discover the students' current understanding of the standards. Teachers should be careful not to assume that their students will know something because of their age or grade level. Students may have algorithmic knowledge without conceptual understanding.

Throughout this study, students used manipulatives to support their understanding of various concepts. However, on the post interview, students were not provided with manipulatives since they were not used on the initial interview. Had students been able to use the manipulatives, they may have been able to demonstrate a better understanding of the material. The assessment given should be aligned with the method of instruction.

References

Ashlock, R.B. (2010). *Error patterns in computation: Using error patterns to help each student* learn (10th ed.). Upper Saddle River, NJ: Pearson.

Carpenter, T.P., Fennema, E., Franke, M.L., Levi, L., & Empson, S.B. (2015). *Children's mathematics: Cognitively guided instruction* (2nd ed.). Portsmouth, NH: Heinemann.

Confrey, J., Nguyen, K.H., Lee, K., Panorkou, N., Corley, A.K., & Maloney, A.P. (2012). *TurnOnCCMath.net: Learning trajectories for the K-8 Common Core Math Standards.* Retrieved from https://www.turnonccmath.net

Durkin, K., & Rittle-Johnson, B. (2014). Diagnosing misconceptions: Revealing changing decimal fraction knowledge. *Learning and Instruction*, 22(3), 21-29.

National Research Council. (2001) *Adding it Up: Helping Children Learn Mathematics.* J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Reys, R.E., Lindquist, M., Lambdin, D.V., & Smith, N.L. (2014). *Helping children learn mathematics* (11th ed.). Hoboken, NJ: Wiley.