The Thirty-Seventh Annual
Eastern Shore High School Mathematics Competition

April 27, 2022

Team Contest Exam

Instructions

Answer as many questions as possible in the time provided. To receive full credit for a correct solution, show all work and provide a clearly written explanation. Solutions will be judged based on correctness, completeness and clarity. (Little credit, if any, will be given for a solution consisting of just a number or a single sentence.) Calculators are allowed **only** on the team contest exam.

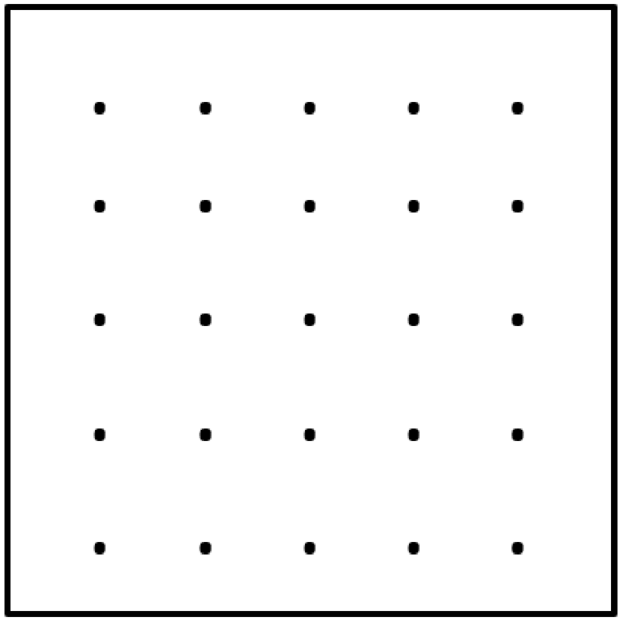
All work and answers must be written on the provided sheets of plain white paper. Use only one side of each sheet of paper, and start each new problem on a new sheet of paper. Write the name of the school which you are representing at the top of each sheet that you turn in for scoring.

At the start of the team round, your team will receive a copy of only Problem 1. Your team must submit a response to Problem 1 within the first 15 minutes of the team round time interval.

When you submit your response for Problem 1, you will receive a copy of Problem 2 and a copy of Problem 3. Your team will then have the time remaining in the team round to complete a response for each problem.

Note: if your team completes Problem 1 before the end of the allotted time, you may submit it and receive copies of Problem 2 and Problem 3 in advance.

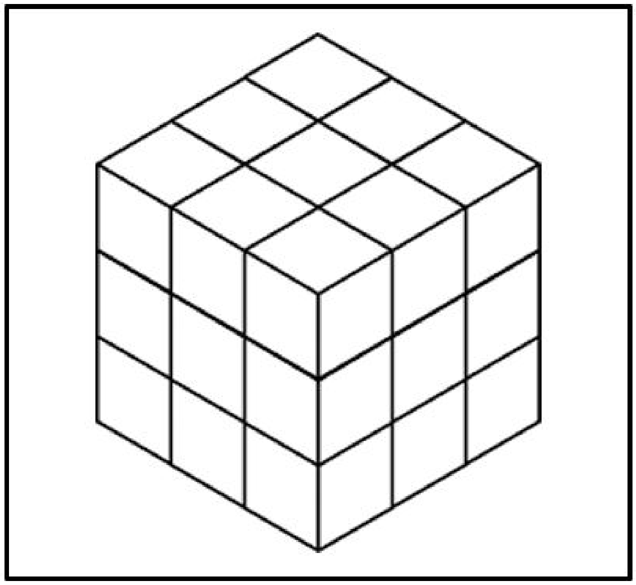
1. Consider a 5×5 rectangular grid where the distance between any two consecutive horizontal or vertical lattice points on the grid is one unit. If two of these points are used as endpoints of a segment, segments of various lengths can be drawn. Draw pentagon $ABCDE$ on the 5×5 rectangular grid that has a perimeter of $9 + \sqrt{2} + \sqrt{13}$ units. State the length of each side of the pentagon you draw.



2. **Task A:** You have six chips and three cups. Three of the chips are Color 1 and three of the chips are Color 2. Place one Color 1 chip and one Color 2 chip in the first cup, two Color 1 chips in the second cup, and two Color 2 chips in the third cup. If you randomly select one cup and remove a Color 1 chip, what is the probability the remaining chip in the cup is also Color 1? Express your answer as a fraction and provide a clear explanation of your solution procedure.

   Task B: You have three cubes numbered 1-6. Each cube is a different color and balanced. If the three cubes are rolled once, what is the probability the sum of the numbers (on top) is less than or equal to 10? Since the cubes are different colors, consider the order of the addends to be important. e.g., If the cubes are blue (B), green (G), and yellow (Y) and the top numbers are $N_B = 4$, $N_G = 5$, and $N_Y = 6$, then $4 + 6 + 5$, and $5 + 6 + 4$ are to be considered as two different ways the sum is equal to 15. Express your answer as a fraction (not as a decimal) and provide a clear explanation of your solution procedure.

3. A “D by D by D” large cube has been painted in each part of this problem. However, the dimensions of the large cubes may or may not be the same. Each face of the large cube has been painted and no paint has seeped inside. The “D by D by D” large cube is made up of small congruent unit cubes. i.e., it is not hollow. e.g., If $D = 3$, a “3 by 3 by 3” large cube has been painted.



Task A: A “5 by 5 by 5” large cube has been painted. How many small cubes have (a) exactly 0 sides painted, (b) exactly 1 side painted, (c) exactly 2 sides painted, (d) exactly 3 sides painted?

Task B: If a “D by D by D” large cube has been painted and exactly two thousand seven hundred forty-four of the small congruent cubes have no sides painted, what is the value of D? Provide justification for your answer.

Task C: If a “D by D by D” large cube has been painted and exactly three thousand four hundred fifty-six of the small congruent cubes have exactly one side painted, what is the value of D and how many of the small congruent cubes have no sides painted? Provide justification for your answer.