The Thirty-Fourth Annual
Eastern Shore High School Mathematics Competition

November 9, 2017

Individual Contest Exam

Instructions

There are twenty problems on this exam. Select the best answer for each problem.

Your score will be the number of *correct* answers that you select.

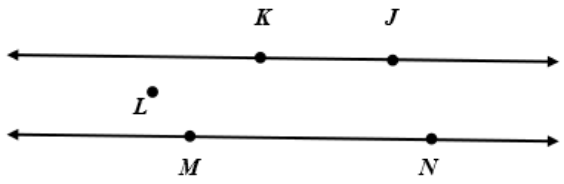
There is no penalty for incorrect answers.

The use of a calculator is **not** permitted on this exam.

No computational work is required for any of your multiple choice responses.

However, in the event of tie scores, after the multiple choice responses have been checked for problems 1-20, the responses and/or written computational work on the enclosed form for problems #18, #19 and #20 will then be used as tiebreakers.

1. In the figure below point L is between $\overleftrightarrow{JK}$ and $\overleftrightarrow{MN}$ and $\overleftrightarrow{JK} \parallel \overleftrightarrow{MN}$. If $s = \text{m}\,\angle JKL + \text{m}\,\angle KLM + \text{m}\,\angle LMN$, then



a. $s < 360^o$ b. $s > 360^o$ c. $s = 360^o$ d. $360^o < s < 540^o$ e. cannot be determined

2. A central angle, θ, in a circle of radius 10 inches intercepts an arc of length 40 inches. What is the radian measure of θ?

a. 0.25 radians
b. 2 radians
c. 3.5 radians
d. 4 radians
e. cannot be determined from the given information

3. If $\ln(x+1) + 2\ln(x+2) - \ln\left((x+2)\left(x^2+5x+4\right)\right) = \ln\left(\frac{2}{3}\right)$, then

a. $x = \dfrac{1}{2}$ b. $x = \sqrt{2}$ c. $x = 2$ d. $x = 2\sqrt{2}$ e. $x = 4$

4. Let $f(x) = ax + b$ and $h(x) = x^2$. Find all ordered pairs (a, b) so that $f(h(x)) = h(f(x))$ for all x.

a. (0,0)
b. (1,0) and (0,1)
c. (1,0), (0,1), and (1,1)
d. (0,0), (1,0), and (0,1)
e. (0,0), (1,0), (0,1), and (1,1)

5. $\tan\left(\text{Sin}^{-1}(x)\right)$ is equal to

a. $\dfrac{x}{\sqrt{1-x^2}}$
b. $\pm\dfrac{x}{\sqrt{1-x^2}}$
c. x
d. $\sqrt{1-x^2}$
e. $\pm\sqrt{1-x^2}$

6. Which of the following represents the set of elements in X or Y or Z but not all three simultaneously? (Note that A' denotes the complement of the set A.)

I: $(X \cup Y \cup Z) \cap (X \cap Y \cap Z)'$

II: $(X \cap (Y \cap Z)') \cup (Y \cap (X \cap Z)') \cup (Z \cap (Y \cap X)')$

III: $(X \cap (Y' \cup Z')) \cup (Y \cap (X' \cup Z')) \cup (Z \cap (Y' \cup X'))$

IV: $(X \cap (Y' \cap Z')) \cup (Y \cap (X' \cap Z')) \cup (Z \cap (Y \cap X)')$

V: $(X \cap (Y \cup Z)') \cup (Y \cap (X \cup Z)') \cup (Z \cap (Y \cup X)')$

a. I only
b. I, IV and V
c. I, II and III
d. I, II and IV
e. none of the above

7. A sample consists of four test scores: 85, 72, 91, and 78. Suppose that one more test score is selected. For which value of the fifth test score is the new sample mean equal to the new median?

a. 64 b. 78 c. 81 d. 99 e. both a and d

8. A group of students at an event is made up of 28 females and 22 males. Twelve of the males are math majors and ten are computer science majors. Twenty of the females are computer science majors and the remaining are math majors. If one of these students is selected at random, what is the probability that the student is a math major if it is known that the student is female?

a. $\frac{8}{50}$ b. $\frac{8}{28}$ c. $\frac{20}{50}$ d. $1 - \frac{12}{22}$ e. $\frac{1}{2}$

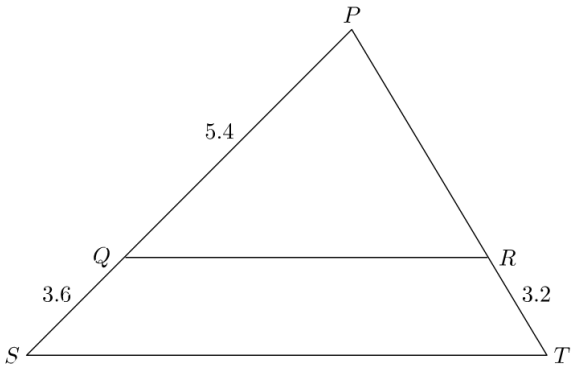
9. The digits 1, 2, 3, 4, and 5 are shuffled and a five-digit number is made. How many of the numbers made this way are greater than 30,000?

a. $4!$ b. $5!$ c. $2 \cdot 4!$ d. $2 \cdot 5!$ e. $3 \cdot 4!$

10. Gina and Jim divide a large number of candies as follows. Gina takes one, and Jim takes one. Then Gina takes two, and Jim takes one. Then Gina takes three, and Jim takes one. This pattern continues until Gina takes 100 candies and Jim then takes the last one. (So Jim has a total of 100 candies, and Gina has significantly more.) How many more candies does Gina have than Jim?

a. 4000 b. 4950 c. 5000 d. 5050 e. 5150

11. In the image shown below, QR is parallel to ST. What is the length of PR?



a. 2.13 b. 4.8 c. 5 d. 6.075 e. 6.5

12. Which one of the following has the value of 2017?

a. $2018^{\log_2(2017-2^{11}+2^4+2^3+2^2+2^1+2^0+2^0)}$

b. $2017^{\log_2(2018-2^{11}+2^4+2^3+2^2+2^1+2^0)}$

c. $2017^{\log_2(2017-2^{11}+2^4+2^3+2^2+2^1+2^1+2^0)}$

d. $(\log_2(2017-2^{11}+2^4+2^3+2^2+2^1+2^1+2^0))^{\log_{2017}(2017^{2018})}$

e. $(\log_2(2017-2^{11}+2^4+2^3+2^2+2^1+2^1+2^0))^{\log_{2018}(2018^{2017})}$

13. $\triangle KLM$ has vertices $K(11,9)$, $L(2,0)$, and $M(1,7)$. $\triangle K'L'M'$ is the image of $\triangle KLM$ after two transformations – a reflection in the y-axis followed by a 90^o counterclockwise rotation about the origin. If X_S is the sum of the x-coordinates of $\triangle K'L'M'$ and Y_S is the sum of the y-coordinates of $\triangle K'L'M'$, then

a. $X_S > Y_S > 0$ b.$Y_S > X_S > 0$ c.$X_S < Y_S < 0$ d. $Y_S < X_S < 0$ e. $X_S = Y_S = 0$

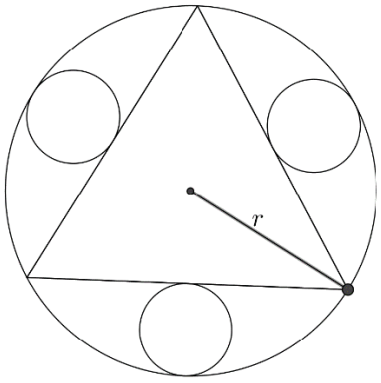
14. A collection of candy bars can be equally distributed among 14, 20, or 105 people such that no candy bars remain. Let X denote the smallest possible number of candy bars in such a collection. Which of the following inequalities is true?

a. $X < 200$ b. $200 \leq X < 400$ c. $400 \leq X < 600$ d. $600 \leq X < 800$ e. $800 \leq X$

15. A bouncy rubber ball is dropped from a height of 16 feet. When the ball hits the ground, it bounces back up $\frac{3}{4}$ of the distance it fell. If the ball repeats this pattern forever, what is the total distance the ball will travel?

a. 48 feet b. 64 feet c. 112 feet d. 128 feet e. infinitely many feet

16. An equilateral triangle is inscribed in a circle of radius r. Between each of the sides and the circumference a circle is drawn tangent to each. What is the sum of the areas of these three circles?



a. $\frac{3}{4}\pi r^2$ b. $\frac{3}{16}\pi r^2$ c. $\frac{9}{4}\pi r^2$ d. $\frac{9}{16}\pi r^2$ e. none of these

17. Suppose it is 11:00 AM right now. What time will it be 2017 hours from now?

a. 4:00 AM b. 7:00 AM c. 12:00 Noon d. 6:00 PM e. none of these

18. Ruby doesn't like grapes, so while no one was looking, she tried to give her grapes to a few of her princess dolls. She always likes to be fair, so she made sure to share the grapes equally among her dolls. However ...

- When she tried to share them equally among 3 dolls, there was 1 grape left over. So, she brought another doll to the table ...
- When she tried to share them equally among 4 dolls, there was still 1 grape left over. So, she went and found yet *another* doll
- When she tried to share them equally among 5 dolls, there was, once again, 1 grape left over.

At this point, she gave up, dejectedly ate one grape herself (she doesn't like to be wasteful), and then shared the rest equally among the 5 dolls.

Assuming she started with fewer than 100 grapes, how many grapes did each of Ruby's five princess dolls end up with?

a. 6 b. 8 c. 9 d. 10 e. 12

19. Martian potatoes are 99% water. Suppose we have 50 pounds of Martian potatoes. If the potatoes are dried so that they are now only 98% water, the new total weight of the potatoes falls in what range?

a. 20 lbs. to 23 lbs.
b. 24 lbs. to 27 lbs.
c. 28 lbs. to 31 lbs.
d. 32 lbs. to 35 lbs.
e. none of these

20. Let 1, 3, 5, ... and 9, 13, 17, ... be two arithmetic sequences, and consider just the first 2017 terms of each sequence. How many numbers do the two sequences have in common?

a. 109 b. 351 c. 621 d. 1007 e. 1344