

# The Thirty-Second Annual Eastern Shore High School Mathematics Competition

November 5, 2015

## Team Contest Exam

### Instructions

Answer as many questions as possible in the time provided. To receive full credit for a correct solution, show all work and provide a clearly written explanation. Solutions will be judged based on correctness, completeness and clarity. (Little credit, if any, will be given for a solution consisting of just a number or a single sentence.) Calculators are allowed **only** on the team contest exam.

All work and answers must be written on the provided sheets of plain white paper. Use only one side of each sheet of paper, and start each new problem on a new sheet of paper. Write your team name (that is, the name of the school which you are representing) at the top of each sheet that you turn in for scoring.

**At the start of the team round, your team will receive a copy of only Problem 1. Your team must submit a response to Problem 1 within the first 15 minutes of the team round time interval.**

**When you submit your response for Problem 1, you will receive a copy of Problem 2 and a copy of Problem 3. Your team will then have the time remaining in the team round to complete a response for each problem.**

Note: if your team completes Problem 1 before the end of the allotted time, you may submit it and receive copies of Problem 2 and Problem 3 in advance.

1. You have 3 “color A” snap cubes, 3 “color B” snap cubes and 3 cups.
  - Place exactly two snap cubes in each cup, such that the contents of each cup are different from the contents of the other cups, and label each cup according to the colors of the snap cubes in the respective cups.
  - Now, switch the labels so that each cup is mislabeled.
  - Without seeing the contents of either cup draw 1 snap cube from a cup of your choice. Continue this process until each cup can be labeled correctly.
  - Determine the minimum number of draws required to do this. Provide a clear, concise and complete discussion of your procedure.

**Provide explanations for each problem. No credit will be given for correct numerical answers without explanations.**

2. Suppose you have two coins in your pocket: a fair coin and a two-headed coin. You select one of the coins at random. When you flip the coin, it shows heads.
  - a. What is the probability that the coin you selected is the fair coin?
  - b. Assume you flip the same coin again and it shows heads. What is the probability that the coin you selected is the fair coin now?
  - c. Assume you flip the same coin again and this time it shows tails. What is the probability that the coin you selected is the fair coin now?
  - d. Suppose you had flipped the coin  $n$  times, for some positive integer  $n$ , and each time the coin showed heads. What is the probability that the coin is fair?

3. In a particular book, some consecutive page numbers have been circled. The sum of the circled page numbers is  $s$ , where  $s$  is some whole number.

The *trivial solution* to this problem is to say that only one page number,  $s$ , is circled.

- a. Explain why the number of circled page numbers cannot be 2 if  $s$  is even.
- b. Explain why there is a solution other than the trivial solution when  $s$  is an odd number greater than 1.
- c. Find a solution other than the trivial solution when  $s = 500$ . Show all of your work.
- d. Find a value of  $s$ , where  $s > 50$ , for which there are at least two solutions other than the trivial solution. Show all of your work.