The Twenty-Sixth Annual

Eastern Shore High School Mathematics Competition

November 12, 2009

Team Contest Exam

Instructions
Answer as many questions as possible in the time provided. To receive full credit for a correct solution, show all work and provide a clearly written explanation. Solutions will be judged based on correctness, completeness and clarity. (Little credit, if any, will be given for a solution consisting of just a number or a single sentence.)

All work and answers must be written on the provided sheets of plain white paper. Use only one side of each sheet of paper, and start each new problem on a new sheet of paper.

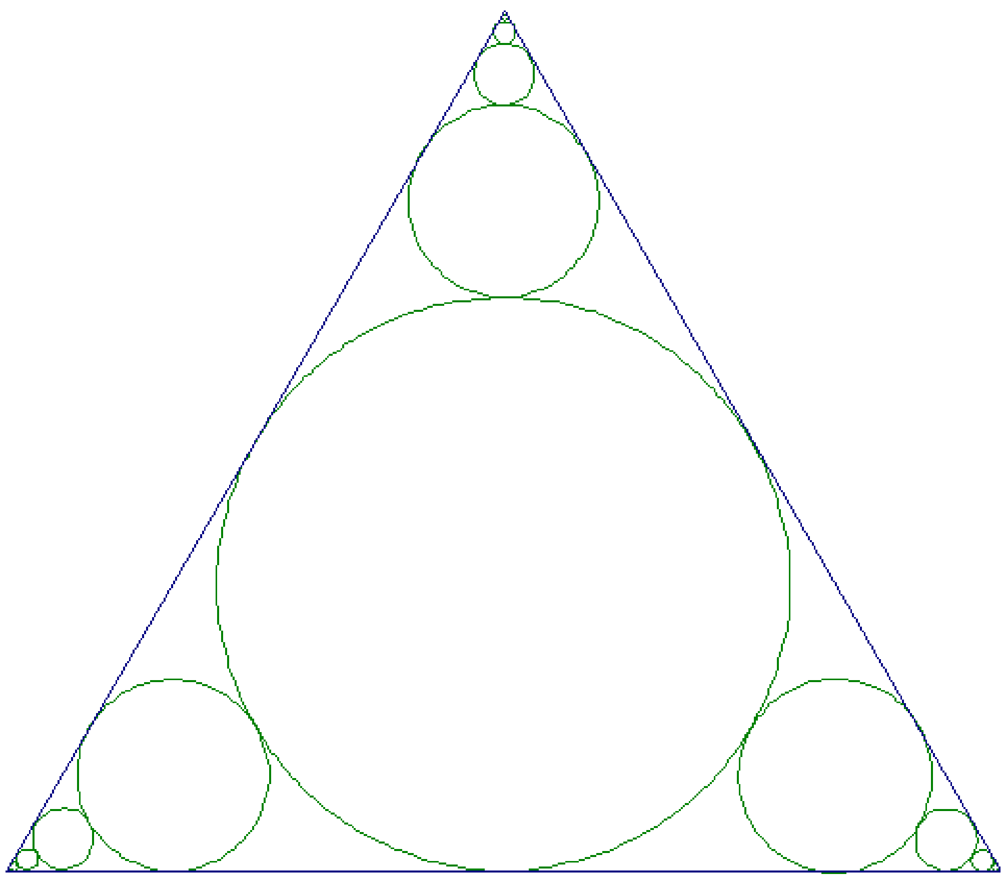
1. Andy and Bob are 30 km apart. They start to walk toward each other at the same time. Andy walks with a constant speed of 4 km per hour. During the first hour, Bob walks with a speed of 2 km per hour. Thereafter, at the beginning of each hour, Bob increases his speed by k km per hour - for example, during the second hour Bob walks with a constant speed of $2 + k$ km per hour, during the third hour Bob walks with a constant speed of $2 + 2k$ km per hour, and so on.

   (a) After how many hours will Andy and Bob reach each other if $k = 1$?

   (b) After how many hours will Andy and Bob reach each other if $k = 0.5$?

   (c) After how many hours will Andy and Bob reach each other if k is between 0 and 4? Write your answer as a function of k.

2. A circle of radius 1" is inscribed in an equilateral triangle. A smaller circle is then inscribed near each vertex of the triangle, tangent to the first circle and to the two sides of the triangle. The process is continued in this way with smaller and smaller circles, as demonstrated in the diagram below. What is the sum of the circumferences of *all* of the circles (not just the few circles shown in the diagram) that will be constructed in this way?



3.

has dog? / # siblings	Yes	No
None	15	9
1 or 2	21	8
more than 2	24	13

Ninety students at a local school were polled; each was asked “Do you own a dog?” and “How many siblings (brothers or sisters) do you have?” The results are tabulated in the table shown above. Determine whether each of the following statements is true or false for a randomly selected student from this class. Explain your conclusion for each answer.

(a) A student who does not own a dog is less likely to be an only child (no brothers or sisters) than a student who does own a dog.

(b) There is at least a 50% chance that a student will own a dog and have at least one sibling.

(c) A student with more than 2 siblings is more likely to own a dog than a student with 1 or 2 siblings.

4. Verify the following algebraic identity for all values of a and b such that $a \neq 0$ and the left-hand side of the identity is defined:

$$\left(\frac{1}{2a-b}+\frac{3b}{b^2-4a^2}-\frac{2}{2a+b}\right)\div\left(\frac{4a^2+b^2}{4a^2-b^2}+1\right)=-\frac{1}{4a}$$

5. Let $\mathcal{S}$ denote the set of nine-digit numbers whose digits are 1, 2, 3, 4, 5, 6, 7, 8, and 9, such that each of these digits occurs exactly once. For example, $\mathcal{S}$ would include the numbers 123456789, 142537698, and 798432516.

For each of the following questions, assume that we are selecting a number from $\mathcal{S}$ at random. Write each answer as a decimal rounded to the nearest thousandth.

(a) What is the probability that the number will be even?

(b) What is the probability that the number will be greater than 300000000?

(c) What is the probability that the first, fifth, and ninth digits will all be odd?

(d) What is the probability that the sum of the number's first two digits will be equal to the number's last digit?