

The Twenty-Fifth Annual

Eastern Shore High School Mathematics Competition

November 12, 2008

Team Contest Exam

Instructions

Answer as many questions as possible in the time provided. To receive full credit for a correct solution, show all work and provide a clearly written explanation. Solutions will be judged based on correctness, completeness and clarity. (Little credit, if any, will be given for a solution consisting of just a number or a single sentence.)

All work and answers must be written on the provided sheets of plain white paper. Use only one side of each sheet of paper, and start each new problem on a new sheet of paper.

1. We have provided you with five cards, each with a different symbol on it. Two cards have the symbol “5” on them, two have the symbol “2”, and the fifth card has the symbol “=” on it.

On the attached sheet, there are boxes labeled “A,” “B,” “C,” “D,” and “E.” In the following instructions, all references to boxes – such as “place a card on box A” – refer to the boxes on the attached sheet.

Part (a)

Instructions:

Place the “=” card face up on box C. Turn the other four cards face down.

Randomly select one of the four remaining cards and place it face up on box A.

Randomly select one of the three remaining cards and place it face up on box B.

Randomly select one of the two remaining cards and place it face up on box D.

Place the remaining card face up on box E.

If the cards now form the equation $5^2 = 25$, then make a tally mark in the “Correct Sequence” row in the table below. Otherwise, make a tally mark in the “Incorrect Sequence” row.

| | Tally | Total |
|--------------------|-------|-------|
| Correct Sequence | | |
| Incorrect Sequence | | |

Repeat the above instructions until you have made a total of at least 30 tally marks.

i) According to your experiment, what is the empirical probability that the cards selected at random will be selected in the correct sequence to form the equation $5^2 = 25$?

ii) What is the theoretical probability of the cards being selected in the correct sequence to form the equation $5^2 = 25$? Explain your answer, and show all of your work.

Part (b)

Instructions:

Turn all five cards face down.

Randomly select one of the cards and place it face up on box A.

Randomly select one of the four remaining cards and place it face up on box B.

Randomly select one of the three remaining cards and place it face up on box C.

Randomly select one of the two remaining cards and place it face up on box D.

Place the remaining card face up on box E.

If the cards now form the equation $5^2 = 25$, then make a tally mark in the “Correct Sequence” row in the table below. Otherwise, make a tally mark in the “Incorrect Sequence” row.

| | Tally | Total |
|--------------------|-------|-------|
| Correct Sequence | | |
| Incorrect Sequence | | |

Repeat the above instructions until you have made a total of at least 30 tally marks.

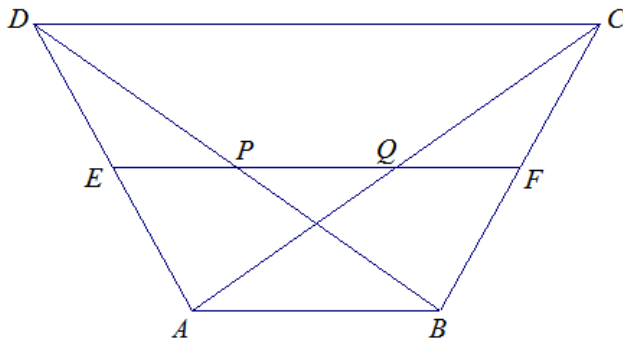
i) According to your experiment, what is the empirical probability that the cards selected at random will be selected in the correct sequence to form the equation $5^2 = 25$?

ii) What is the theoretical probability of the cards being selected in the correct sequence to form the equation $5^2 = 25$? Explain your answer, and show all of your work.

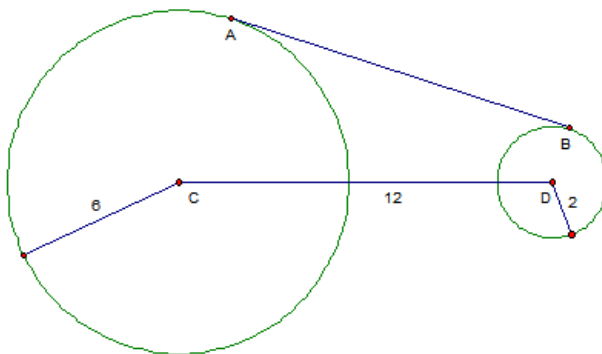
2. The quadrilateral $ABCD$ (shown below) is a trapezoid, with \overleftrightarrow{AB} parallel to \overleftrightarrow{DC} . Points E and F are the midpoints of \overline{AD} and \overline{BC} , respectively. Line segments \overline{DB} and \overline{EF} intersect at P , and \overline{AC} and \overline{EF} intersect at Q .

(a) If $|AB| = 18$ and $|DC| = 28$, then what is the value of $\frac{|PQ|}{|EF|}$?

(b) More generally: if $|AB| = m$ and $|DC| = n$, where m, n are both positive and $m < n$, then then what is the value of $\frac{|PQ|}{|EF|}$, in terms of m and n ?



3. Two circles, with radii 6 and 2, are coplanar, and their centers are 12 units apart, as shown below. Line \overleftrightarrow{AB} is a common external tangent line to the two circles, where A and B are the points at which this tangent intersects the larger circle and the smaller circle, respectively. Find the length of line segment \overline{AB} .



4. Let $p = \log_4 3$ and $q = \log_3 5$. Find the *exact* value of each of the following in terms of p and q .
- (a) $\log_4 5$
- (b) $\log_{10} 5$