

The Twenty-Fifth Annual

Eastern Shore High School Mathematics Competition

November 12, 2008

Individual Contest Exam

Instructions

Select the best answer for each question on this exam.

Your score will be the number of *correct* answers that you select.

There is no penalty for incorrect answers.

The use of a calculator is *not* permitted on this exam.

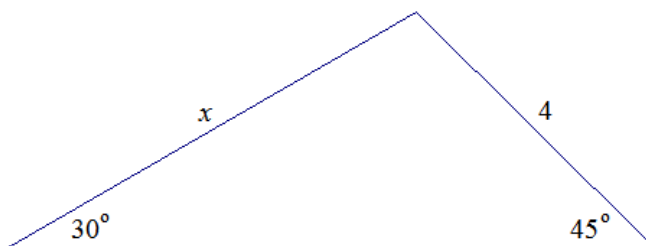
In the event of tie scores, questions #18, #19 and #20 will be used as tiebreakers.

1. A produce stand sells apples for \$2 per dozen and oranges for \$3 per dozen. If a company wants to buy 15 dozen pieces of fruit for its picnic, and it has to spend \$40, then how many dozen apples and oranges should it buy?
- (a) 8 dozen apples and 8 dozen oranges
 (b) 10 dozen apples and 5 dozen oranges
 (c) 11 dozen apples and 6 dozen oranges
 (d) 5 dozen apples and 10 dozen oranges
 (e) 8 dozen apples and 7 dozen oranges
2. Solve for x : $\sqrt{x^4 + 4x^2 + 4} = 3x$.
- (a) $x = \pm 1, \pm 2$ (b) $x = 0, 2/3$ (c) $x = 1$ (d) $x = 1, 2$ (e) No real solutions exist
3. Assume $x > 1$, $y > 1$ and $z > 1$. If $\frac{1}{4} \log_y x = \log_y z$, then which of the following *must* be true?
- (a) $x = z$ (b) $z^2 = x$ (c) $x^4 = z$ (d) $z^{1/4} = x$ (e) None of these
4. A list of five numbers has exactly one mode, which is 8. If four of the five numbers in the set are 3,3,8,8, then what is the mean of all five numbers in the set?
- (a) 3 (b) 5.5 (c) 6 (d) 8 (e) Cannot be determined from the information given
5. A high school class consists of 18 girls and 15 boys. Of the 18 girls, 6 are planning to go to college. Of the 15 boys, 9 are planning to go to college. If a student is randomly selected from this class, what is the probability the student is a girl who plans to go to college?
- (a) $6/33$ (b) $6/18$ (c) $6/9$ (d) $522/524$ (e) None of these
6. Which of the following is/are equal to $(\log_8 64 \cdot \log_{11} 121 \cdot \log_{10} 100 \cdot \log_{12} 144)^2$?
- I. $\log_8 (8^{256})$ II. $\log_4 (2^{512})$ III. $\log_{16} (256^8)$
- (a) I (but not II or III) (b) I and II (but not III) (c) II (but not I or III)
 (d) II and III (but not I) (e) I, II and III
7. Which of the following conditions imply $B = C$?
- I. $A \cup B = A \cup C$
 II. $A \cap B = A \cap C$
- (a) I only (b) II only (c) Both I and II (d) Neither I nor II
8. Groups A, B and C have 10, 12 and 14 members, respectively. The total population is 25. If 6 members belong to both A and B, 4 members belong to both A and C, and 5 members belong to both B and C, then how many members belong to all three groups?
- (a) 0 (b) 4 (c) 5 (d) 6 (e) 15

9. John sells whatchamacallits for a reasonable price, and he sold so many on Monday that he raised his selling price 20% on Tuesday. He sold none. So he lowered his price 10% for Wednesday. When he still didn't sell many, he once again lowered his price 10% for Thursday. He sold as many whatchamacallits as he had on Monday. How does John's price for whatchamacallits on Monday compare to their price on Thursday?

- (a) Monday's price is the same as Thursday's price.
- (b) Monday's price is higher than Thursday's price.
- (c) Monday's price is lower than Thursday's price.
- (d) At least two of the answers (a), (b), or (c) could be true, depending upon Monday's price.
- (e) It is impossible to determine the relationship between Monday's and Thursday's prices.

10. Using the information given in the figure below, solve for x .



- (a) $\sqrt{2}$
- (b) $\sqrt{6}$
- (c) $2\sqrt{6}$
- (d) $4\sqrt{2}$
- (e) 8

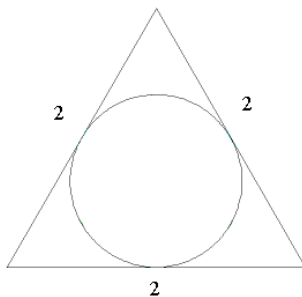
11. Let the operation $*$ be defined as follows:

$$\text{For all } a > 0 \text{ and } b > 0, a * b = a^b + b^a.$$

For this operation, which of the following is *not* true?

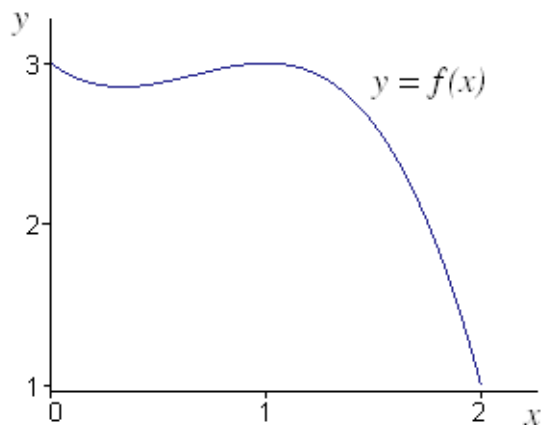
- (a) $2 * 3 = 17$
- (b) $3 * 4 = 4 * 3$
- (c) $1 * 2 = 2 * 1$
- (d) $1 * x = x + 1$ for all $x > 0$
- (e) $1 * x = x$ for all $x > 0$

12. A circle is inscribed in an equilateral triangle of side length 2, as shown in the diagram below. What is the area of the inscribed circle?



- (a) $\pi/4$
- (b) $2 - \pi/3$
- (c) $\pi/3$
- (d) $2 - \pi/4$
- (e) $\pi/2$

13. Based on the graph shown below, which inequality is true?



- (a) $f(0) > 2 > f(2)$
 - (b) $f(0) > f(2) > 2$
 - (c) $2 > f(0) > f(2)$
 - (d) $2 > f(2) > f(0)$
 - (e) $f(2) > f(0) > 2$
14. The sixth term of a geometric series is 3072. The third term is 48. Find the first term.
- (a) $\frac{3}{4}$
 - (b) 3
 - (c) 4
 - (d) 12
 - (e) None of these
15. If $\sin \theta = \frac{3}{5}$ and θ is in the second quadrant, then what is the value of $\sin 2\theta$?
- (a) $-\frac{24}{25}$
 - (b) $-\frac{9}{25}$
 - (c) $\frac{9}{25}$
 - (d) $\frac{24}{25}$
 - (e) $\frac{6}{5}$
16. What is true about the two circles whose equations are $(x - 1)^2 + (y + 2)^2 = 5$ and $x^2 + y^2 + 2y = 0$?
- a) They are tangent to one another, and one is contained inside the other.
 - b) They are tangent to one another, but neither is contained inside the other.
 - c) They intersect at exactly two points.
 - d) They do not intersect.

17. How many whole numbers between 1000 and 2008 include a 2 and a 5 as consecutive digits, in that order? (For example, we *would* count 1258, since the 2 and 5 appear consecutively. We would *not* count 1528, since the digits 2 and 5 are consecutive but in the wrong order. We would *not* count 1285, since the digits 2 and 5 appear but are not consecutive.)

- (a) 10 (b) 20 (c) 25 (d) 30 (e) 100

18. (Tiebreaker #1) An adelsman is a person who always tells the truth. A kurs is a person who always lies.

A says: "B is an adelsman." B says: "A and I are of opposite types."

What are A and B?

- (a) A is an adelsman; B is a kurs.
(b) A is a kurs; B is an adelsman
(c) Both A and B are kurs.
(d) Both A and B are adelsmen.
(e) We don't have enough information to answer the question.

19. (Tiebreaker #2) Find all values of x , $0 \leq x \leq 2\pi$, satisfying the following equation.

$$x \sec x - 2x - 6 \sec x + 12 = 0$$

- (a) 6 (b) $6, \frac{\pi}{4}, \frac{3\pi}{4}$ (c) $6, \frac{\pi}{6}, \frac{11\pi}{6}$ (d) $6, \frac{\pi}{3}, \frac{2\pi}{3}$ (e) $6, \frac{\pi}{3}, \frac{5\pi}{3}$

20. (Tiebreaker #3) If $f(x) = \frac{2}{x^5}$, then what is the value of $f(f(f(f(f(2))))))$?

- (a) 2^{-2604} (b) 2^{-1302} (c) 2^{-1024} (d) 2^{1302} (e) 2^{2604}