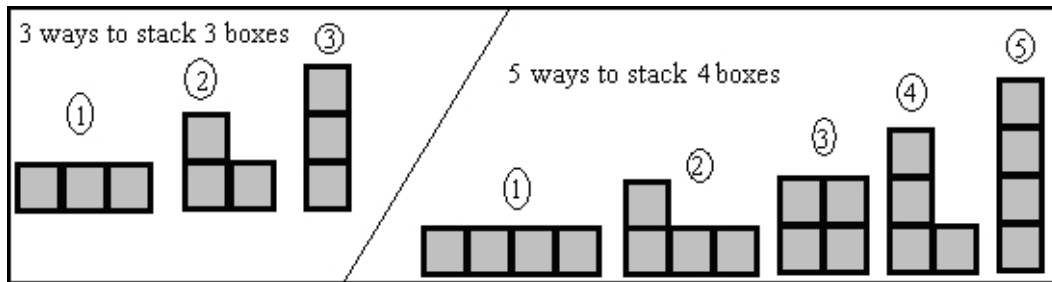


Eastern Shore High School Mathematics Competition
 November 14, 2007
 Team Contest

Answer as many questions as possible in the time provided. To receive full credit for a correct solution, show all work and provide a clearly written explanation. Solutions will be judged based on correctness, completeness and clarity. (Little credit, if any, will be given for a solution consisting of just a number or a single sentence.)

All work and answers must be written on the provided sheets of plain white paper. Use only one side of each sheet of paper, and start each new problem on a new sheet of paper.

1. We will stack boxes in such a way that each “layer” (or row) of boxes has at least as many boxes as the layer above it, and on each “layer” all the boxes must be pushed as far as possible to the left. For example, the diagram below shows all of the possible ways to stack three boxes or four boxes under these rules. Note that there are three different ways to stack three boxes, and there are five different ways to stack four boxes.



- Draw diagrams (similar to the above diagram) for 5 boxes, 6 boxes and 7 boxes. (Use the provided manipulatives to get a feel for this question; make sure you don't miss any possibilities!)
- How many ways are there to stack 5 boxes using an even number of layers? 6 boxes? 7 boxes?
- How many ways are there to stack 5 boxes in such a way that the bottom layer has an even number of boxes? 6 boxes? 7 boxes?
- Based on your answers to (b) and (c), you should be able to conjecture a general connection between the number of ways to stack N boxes using an even number of layers and the number of ways to stack N boxes in such a way that the bottom layer has an even number of boxes. State this conjecture in words, and explain why it will be true for all values of N .

2. $\triangle ABC$ and $\triangle CDA$ are equilateral triangles (sharing side \overline{AC}), point E is the midpoint of side \overline{AB} , and line \overleftrightarrow{DE} intersects side \overline{AC} at point F . The length of side \overline{CD} is $\sqrt{7}$. Find the EXACT length (not just a decimal approximation) of \overline{AF} .

3. A natural number is called a “wonderful number” if each of the following is true:

- It is a multiple of 8
- It has a remainder of 2 when divided by 3
- It has a remainder of 2 when divided by 5

- (a) What is the smallest wonderful number?
- (b) Find all of the wonderful numbers between 0 and 500.
- (c) Find a formula for all wonderful numbers.

4. Mike found a great investment opportunity that will increase in value by 50% each week.

Suppose he invests \$10 today.

(Give whole number answers for questions (a), (b) and (c). For (d), answer in terms of x .)

- (a) After how many weeks will Mike have over one million dollars?
- (b) After how many weeks will Mike have over one billion dollars?
- (c) After how many weeks will Mike have over one trillion dollars?

- (d) After how many weeks will Mike have x dollars, where $x > 10$?