

Eastern Shore High School Mathematics Competition  
November 14, 2007  
Individual Contest

Choose the best answer to each of the following 20 questions. Your score will be the number of correct answers you give out of 20 possible. There is no penalty for incorrect answers. In the event of tie scores, questions #18, #19 and #20 (in that order) will be used as tiebreakers.

1. If  $f(g(x)) = x^4$  and  $f(x) = (x + 1)^2$ , then  $g(x)$  is which of the following?

- (a)  $x^2 - 1$     (b)  $x^2 + 1$     (c)  $x^4 + 1$     (d)  $(x + 1)^4$     (e)  $\frac{x^4}{(x + 1)^2}$

2. A chord of length 10 of the circle  $x^2 + y^2 = 25$  has one of its endpoints at (3,4). What are the coordinates of the chord's other endpoint?

- (a) (-4,3)    (b) (-3,-4)    (c) (-3,4)    (d) (3,-4)    (e) (4,3)

3. Four pieces of luggage were checked at an airline counter. Their weights (in pounds) are 32, 28, 30 and 37. Now, a fifth piece of luggage is checked. The mean and the median weight of the given pieces of luggage are now the same. Which of the following was the weight (in pounds) of the fifth piece of luggage?

- (a) 28    (b) 30    (c) 31    (d) 33    (e) 35

4. Starting with the number 8: first, increase the number by 10%. Next, decrease the resulting number by 10%, then increase that number by 20%, and finally, decrease that number by 20%, finishing with a number, which we'll call  $x$ . Which of the following is true?

- (a)  $7.25 \leq x < 7.5$   
(b)  $7.5 \leq x < 7.75$   
(c)  $7.75 \leq x < 8$   
(d)  $8 \leq x < 8.25$   
(e)  $8.25 \leq x < 8.5$

5. If the point (2,1) is rotated about the origin by 90 degrees in a clockwise direction, the coordinates of its image are which of the following?

- (a) (-1,2)    (b) (1,-2)    (c) (-2,1)    (d) (2,-1)    (e) None of these

6. If  $f(x) = 3^{x/3}$ , then what is the value of  $f\left(\log_6 \frac{1}{216}\right)$ ?

- (a) -27    (b) -3    (c)  $\frac{1}{216}$     (d)  $\frac{1}{27}$     (e)  $\frac{1}{3}$

7. If  $f(x) = \frac{1}{x^3}$ , then  $f(f(f(f(x))))$  is which of the following?
- (a)  $x^3 - 1$     (b)  $1/x^9$     (c)  $x^{81}$     (d)  $1/x^{81}$     (e) None of these
8. Assuming  $\pi \approx 3$ , consider a sphere of radius 5 inscribed in a cube. What is the approximate volume of the region contained *between* the sphere and the cube?
- (a) 300    (b) 400    (c) 500    (d) 600    (e) 700
9. Of thirty students at East Coast High School, 20 are taking math, 15 are taking English, and 8 are taking both. If one of these 30 students is randomly selected, what is the probability that the student is taking neither math nor English?
- (a) 3/30    (b) 8/30    (c) 22/30    (d) 27/30    (e) None of these
10. If  $(2^4)(2^{4n})(4) = 16$ , then what is the value of  $n$ ?
- (a) -2    (b) -1/2    (c) 0    (d) 4    (e) None of these
11. The number 2007 has how many distinct prime factors?
- (a) 1    (b) 2    (c) 3    (d) 4    (e) more than 4
12. A piece of paper is 0.01 inches thick. It is folded in such a way that its thickness is doubled by each fold. If the paper is folded 5 times, how thick is the result (in inches)?
- (a) 0.02    (b) 0.05    (c) 0.10    (d) 0.31    (e) 0.32
13. Find the solution set of the following equation.
- $$10^{\log_{10}(x+1) + 2\log_{10}(x)} - 3^{\log_3(5)} = x^3 + 4x$$
- (a)  $\{-1, 5\}$     (b)  $\{0\}$     (c)  $\{5\}$     (d)  $\{5, 10\}$     (e)  $\emptyset$
14. Suppose the number  $48m25n$  (where  $m$  and  $n$  are unknown digits) is divisible by 9. Evaluate  $m + n$ .
- (a) 2    (b) 8    (c) 9    (d) 18    (e) Cannot be determined from the information given.

15. Charlie and Mac sit down to eat some brown and red M&M's. Initially, Charlie has twice as many brown M&M's as red, while Mac has 6 more red M&M's than twice his number of brown. Mac then gives Charlie 5 red M&M's, after which Charlie has 20 more brown than red, while Mac has 18 more red than brown. How many total M&M's do Charlie and Mac have?
- (a) 67    (b) 75    (c) 127    (d) 132    (e) 137
16. Everyone at the party is wearing a blue shirt. Everyone on the lacrosse team is at the party. Everyone at the party who owns a dog is on the lacrosse team. Jason is wearing a blue shirt, and he owns a dog. Of the following statements, which one(s) *must* be true?
- I. Jason is at the party.  
II. Jason is on the lacrosse team.  
III. If Jason is at the party, then he is on the lacrosse team.
- (a) II and III must be true, but not I.  
(b) I, II and III all must be true.  
(c) Only I must be true.  
(d) Only II must be true.  
(e) Only III must be true.
17. A positive integer  $N$  has a prime factorization given by the product of five distinct primes. How many different factors does  $N$  have?
- (a)  $2^5$     (b) 5    (c)  $5^2$     (d)  $5 \times 5 \times 5 \times 5 \times 5$   
(e) It cannot be determined because the primes are unknown.
18. (First tiebreaker) At bedtime, Homer cuts a pie into three equal pieces. When Homer isn't looking, Bart steals two of the pieces, leaving only one piece for Homer. Grumbling, Homer immediately eats one third of the remaining piece of pie. Exactly one hour later, Homer eats one third of what's left (of his piece). Exactly half an hour after that, Homer eats one third of what's left. Exactly a quarter of an hour after that, Homer eats one third of what's left. Exactly an eighth of an hour after that, Homer eats one third of what's left...  
...and so on. If Homer continues eating in this way, what fraction of the original pie will Homer have eaten by the time Marge gets up the next morning?
- (a) one ninth    (b) one sixth    (c) one fourth    (d) one third    (e) one half

19. (Second tiebreaker) A “perfect number” is a positive integer,  $n$ , that is equal to the sum of all of the factors of  $n$  that are less than  $n$ . For example, the factors of 6 that are less than 6 are 1, 2 and 3; since  $6=1+2+3$ , 6 is a perfect number.

Which of the following statements, if any, are true?

- I. 28 is a perfect number.
- II. 17 is not a perfect number.
- III. 496 is a perfect number.

- (a) Only I is true.
  - (b) I and II are true and III is false.
  - (c) I and III are true and II is false.
  - (d) I, II and III are all true.
  - (e) None of the above combinations is correct.
20. (Third tiebreaker) Suppose an integer sequence of numbers  $a_n, n = 1, 2, 3, \dots$  is defined as follows:

$$a_1 = 1, a_2 = 2, \text{ and } a_{n+1} = 2a_n + a_{n-1} \text{ for } n \geq 2.$$

Assuming the ratio of consecutive terms,  $\frac{a_{n+1}}{a_n}$ , approaches a fixed number  $L$  as  $n$  gets larger without bound, which of the following is true?

- (a)  $L = \sqrt{2}$
- (b)  $L$  is the Golden Ratio
- (c)  $L = 2.5$
- (d)  $L - 2$  is negative
- (e)  $L = \frac{(L - 1)(L + 1)}{2}$