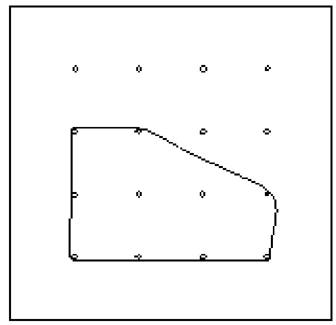
Eastern Shore High School Mathematics Competition
November 8, 2006
Team Contest

Answer as many questions as possible in the time provided. To receive full credit for a correct solution, show all work and provide a clearly written explanation. Solutions will be judged based on correctness, completeness and clarity. (Little credit, if any, will be given for a solution consisting of just a number or a single sentence.)

All work and answers must be written on the provided sheets of plain white paper. Use only one side of each sheet of paper, and start each new problem on a new sheet of paper.

1. Geoboard Polygons



In the geoboard sketch shown to the right, a rubber band has been stretched around some pegs to create a pentagon. The rubber band touches 8 pegs; these eight are the pentagon's *boundary pegs*. In the interior of the pentagon are two pegs not touching the rubber band; these two are its *interior pegs*.

a) On the geoboard we've provided for you, create a polygon with four boundary pegs and one interior peg, a polygon with four boundary pegs and two interior pegs, and a polygon with four boundary pegs and three interior pegs. Calculate the areas of these polygons, and record these areas in a table (as shown).

Number of Interior Pegs	Area (4 Boundary Pegs)
1	
2	
3	

b) Now, create one of each of the following: a polygon with four boundary pegs and two interior pegs, a polygon with five boundary pegs and two interior pegs, and a polygon with six boundary pegs and two interior pegs. Record the areas of of these polygons in a table (as shown).

Number of Boundary Pegs	Area (2 Interior Pegs)
4	
5	
6	

Based on the data you've collected, answer the following questions:

c) Assuming that the area of a polygon with four boundary pegs is a linear function of the number of interior pegs, i, find a formula for the area function, $A(i)$.

d) Assuming that the area of a polygon with two interior pegs is a linear function of the number of boundary pegs, b, find a formula for the area function, $A(b)$.

e) Assuming that the area of a polygon is a linear function of the number of interior pegs, i, and the number of boundary pegs, b, find a formula for the area function, $A(i,b)$.

2. Every Thursday evening, Ed, Fran and Gus get together to watch elbowball on television. Each of the three elbowball fans in this group has exactly three favorite teams, and each pair of fans has exactly one favorite team in common. Show that there must be at least six elbowball teams.

3. Three men and a monkey are on a desert island where they spend the day gathering coconuts. They don't keep an exact count of the coconuts; all they know for sure is that they collected fewer than 100 coconuts in all. The three men agree to divide the coconuts equally in the morning.

   During the night the first man sneaks out to the pile of coconuts and divides it into three equal portions, with one coconut left over - which he gives to the monkey. He hides his portion, then puts the remaining coconuts back into a pile before he returns to bed.

   Even later that night, the second man sneaks out to the pile and divides it into three equal portions, with one coconut left over - which he gives to the monkey. He hides his portion, then puts the remaining coconuts back into a pile before he returns to bed.

   Still later that night, the third man sneaks out to the pile and divides it into three equal portions, with one coconut left over - which he gives to the monkey. He hides his portion, then puts the remaining coconuts back into a pile before he returns to bed.

   In the morning, the three men divide the remaining pile into three equal portions, with one left over - which they give to the monkey.

   How many coconuts were originally gathered? And, how many coconuts did each of the three men receive?

4. In the figure shown below, point A is at the origin, point B is at (4,0), and point C is at (3,2). The triangles $\triangle ACE$, $\triangle CBD$ and $\triangle ABF$ are all equilateral.

   (a) Find the sum of the distances from point (2,1) to each of the vertices of $\triangle ABC$.

   (b) Lines $\overleftrightarrow{AD}$, $\overleftrightarrow{BE}$ and $\overleftrightarrow{CF}$ all intersect at a single point. Find the coordinates of this point.

   (c) Find the sum of the distances from this intersection point to each of the vertices of $\triangle ABC$.