Eastern Shore High School Mathematics Competition
November 8, 2006
Individual Contest

Select the best response for each of the twenty questions. Your score will be the number of questions out of twenty that you answer correctly. There is *no penalty* for incorrect answers, so you should record an answer (even if it's just a guess) for all twenty questions. (In the event of a tie, #18, #19 and #20 will be used as tiebreaker questions.) Diagrams on this exam are *not* necessarily to scale.

1. What is the 2006^{th} term of the arithmetic progression $2006, 2003, 2000, 1997, \ldots$?

   (a) -6009 (b) -4009 (c) -4006 (d) -2006 (e) 0

2. In the xy-plane, consider the following functions on the points (x, y):

   $f((x, y))$ = the reflection of the point (x, y) across the y-axis

   $g((x, y))$ = the reflection of the point (x, y) across the x-axis

   $h((x, y))$ = the reflection of the point (x, y) across the line $x = 3$

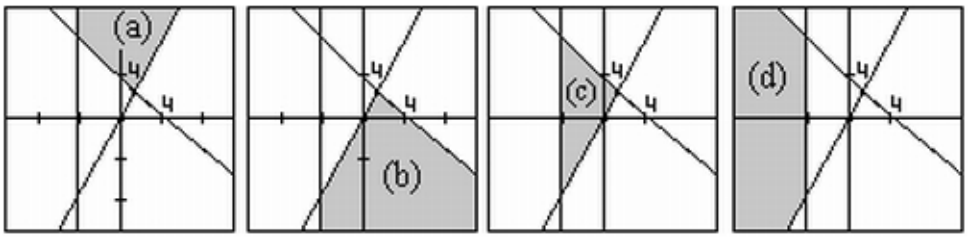
   Given the point (1,2), what is $f(g(h(g(f((1, 2))))))$?

   (a) (-7,-2) (b) (-7,2) (c) (1,2) (d) (-6,2) (e) None of these

3. What is the product of two numbers whose sum is 10 and whose reciprocals add up to 5/12?

   (a) 12/5 (b) 25/6 (c) 16 (d) 24 (e) None of these

4. Which region of the plane satisfies $y > 2x$, $y + x < 4$ and $x + 4 > 0$?



   (e) None of these

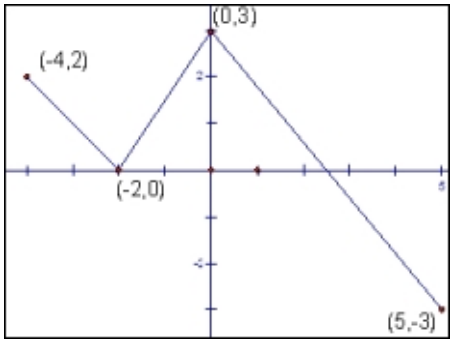
5. Suppose that Norm is 18 years older than Woody, and that, 13 years ago, Norm was three times as old as Woody. How old is Woody now?

   (a) 5 (b) 9 (c) 22 (d) 40 (e) None of these

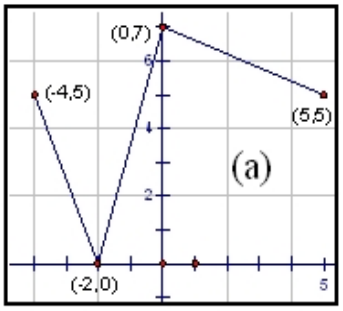
6. A sample consists of four test scores: 85 , 72 , 91 , 78. Suppose one more test score is added to the sample. Find a value of the fifth test score so that the new sample mean is equal to the new median.

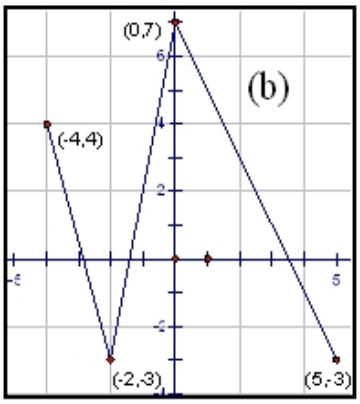
(a) 64 (b) 70 (c) 78 (d) 89 (e) 96

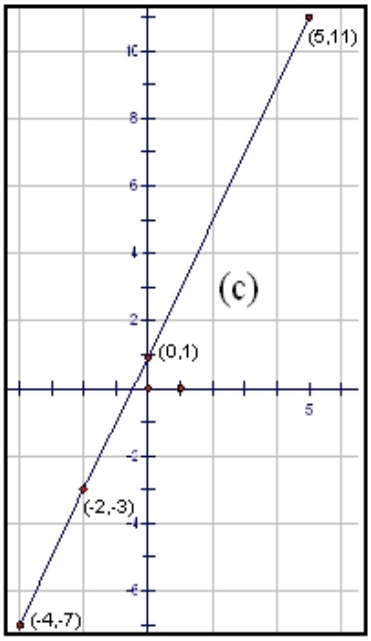
7.

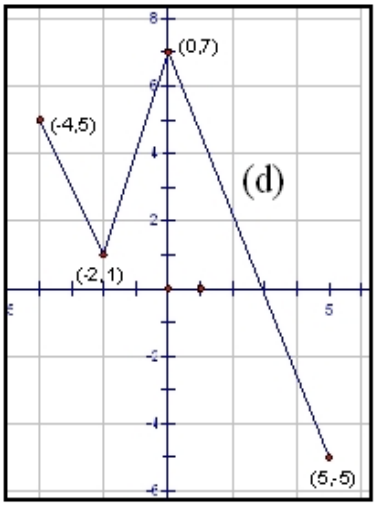


The above picture shows the graph of some function $f(x)$. Which one of the following shows the graph of $2f(x) + 1$?







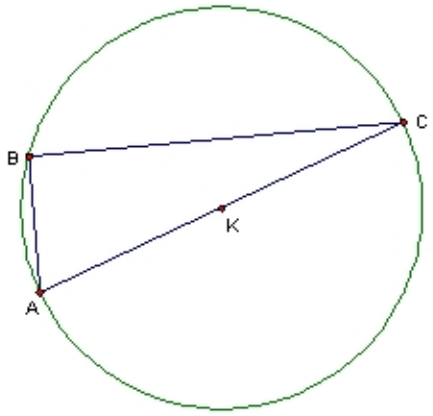


(e) None of these

8. Let P be the set of prime factors of the integer 2004, and let F be the set of the prime factors of 2006. How many elements are in the union of P and F?

(a) 1 (b) 3 (c) 4 (d) 5 (e) 6

9.



In the above diagram, the circle centered at K has radius 10, triangle ABC is a right triangle, AB=10, and AC is a diameter of the circle. Find the measure of the largest acute angle in triangle ABC.

(a) 15° (b) 30° (c) 45° (d) 60° (e) 75°

10. An internet research group chose 80 web sites at random, and searched for each of them on each of three search engines - Gaggle, Yoohoo, and Inquire - to see which sites had been found by each search engine. The results were as follows: 65 of the sites were found by Gaggle, 63 were found by Yoohoo, and 59 were found by Inquire. Further, 54 of the sites were found by both Gaggle and Yoohoo, 48 were found by both Gaggle and Inquire, and 45 were found by both Yoohoo and Inquire. There were 38 sites that were found by all three search engines.

    How many of the chosen web sites were found by at least one of the three search engines?

    (a) 2 (b) 40 (c) 65 (d) 78 (e) 187

11. If P, Q and T are sets, then the statement "x belongs to $(P \cup Q) \cap (T \cap Q)$" is logically equivalent to which of the following?
    (a) x belongs to P and T
    (b) x belongs to T and Q
    (c) x belongs to P and Q but not in T
    (d) x belongs to at least one of the three sets
    (e) x belongs to all three sets

12. A radio station is giving away concert tickets. A list of ten songs will be made available to listeners. Listeners are to call in their pick of three songs. The station will play a random selection of three songs from the list of ten. If a caller matches the three songs to those that are played on the radio (but not necessarily in the same order as played), the caller wins tickets to the concert. What is each caller's probability of winning the tickets?

    (a) 1/1000 (b) 1/720 (c) 1/120 (d) 1/30 (e) 3/10

13. Solve for x:

$$2^{\log_2 e^{\ln 5^{\log_5 7^{\log_7 10^{\log(8x-3)}}}}} = 13$$

(a) $\dfrac{\log_2 13 + 3}{8}$ (b) 1 (c) 2 (d) e (e) $\dfrac{10^{13} + 3}{8}$

14. Assume the following statement is true:

"If the watch costs less than 40 dollars, then Jim will buy it."

Now consider the following three statements:
I. If the watch costs more than 40 dollars, then Jim will not buy it.
II. If Jim will not buy the watch, then the watch costs at least 40 dollars.
III. If Jim will buy the watch, then the watch costs less than 40 dollars.

Which of these three statements must be true?

(a) I only
(b) I and III only
(c) II only
(d) II and III only
(e) None of these statements must be true

15. On his 201st birthday, Jack Skellington decides to put \$1000 in a coffin to save for his 300th birthday party. He plans to save some money every year on his birthday, although in decreasing amounts. For his 202nd birthday, he puts another \$500 in the coffin and for his 203rd he puts in \$250. Jack likes the idea of putting in exactly half the amount he contributed the previous year. If he continues to do this on each birthday up to (and including) his 300th birthday, then how much money (in dollars) will he have saved for his 300th birthday party?

(a) $2000((1/2)^{300})$ (b) $2000((1/2)^{100})$ (c) $300(1-(1/2)^{2000})$ (d) $1000(1-(1/2)^{200})$ (e) $2000(1-(1/2)^{100})$

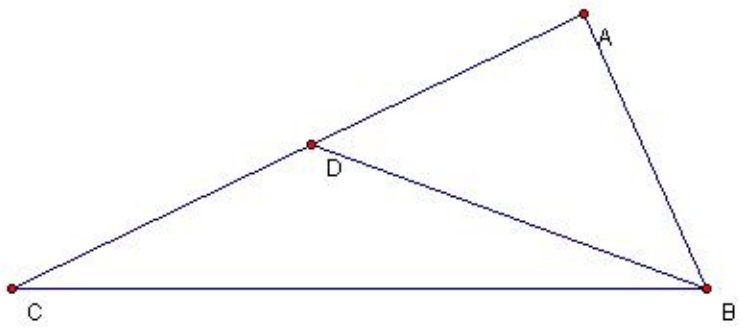
16. How many times does the digit 1 appear in the base two (binary) representation of the decimal number 118?

(a) 3 (b) 4 (c) 5 (d) 6 (e) 7

17. If a, b, c and d are all positive real numbers, then which of the following is equivalent to $\log_d a \cdot \log_c b$?

(a) $\log_a b \cdot \log_c d$ (b) $\log_a d \cdot \log_b c$ (c) $\log_c a \cdot \log_d b$ (d) $\log_c d \cdot \log_b a$ (e) $\log_d a \log_b a$

18. (First tiebreaker)



In the above diagram, $AB = AD$ and $m\angle ABC - m\angle ACB = 30°$. Find $m\angle CBD$.

(a) $15°$
(b) $20°$
(c) $22.5°$
(d) $30°$
(e) $45°$

19. (Second tiebreaker) Let K and L be points on the circle centered at the origin with radius 3. Both points are moving along the circle in the counter-clockwise direction; K is moving with rotational velocity $\pi/3$ and L is moving with rotational velocity $\pi/4$. If K is located at (3,0) and L is at $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$, then how many units of time will elapse before K and L occupy the same location ?

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

20. (Third tiebreaker) Evaluate:

$$\left(2 + \cfrac{3}{4 + \cfrac{3}{4 + \cfrac{3}{4 + \ldots}}}\right)^2.$$

(a) 6 (b) 7 (c) e^2 (d) 8 (e) 25