

Eastern Shore High School Mathematics Competition
November 9, 2005
Individual Contest

Select the best response for each question. The tiebreaker questions are #18, #19 and #20.

1. Which of the following is a solution to the equation $\frac{x(x-3)}{(x-1)(x-2)} = -9$?
(a) 0.5 (b) 1.5 (c) 2.5 (d) 3.5 (e) None of these
2. Which of the following is equivalent to $x - 2(x + 3(x - 4(x - y)))$?
(a) $-2x - y$ (b) $17x - 24y$ (c) $33x + 12y$ (d) $x^3 + x^2 - 10x + 4y$ (e) $-24x^4y$
3. On three math tests, Nathan's mean score was 80, while on five reading tests, his mean score was 90. What was Nathan's mean score over all eight tests?
(a) 21.25 (b) 83.75 (c) 85 (d) 86.25 (e) Cannot be determined
4. You are given a regular pentagon in the xy plane. The endpoints of one of its sides have coordinates (0,3) and (3,7). What is the perimeter of the pentagon?
(a) 4 (b) 5 (c) 20 (d) 25 (e) 125
5. Let x and y be numbers such that their sum is equal to their product. Which of the following statements *must* be true?
I. $x \neq 1$ and $y \neq 1$. II. $\frac{y}{y-1} + \frac{x}{x-1} = xy$. III. $(x-1)(y-1) = 1$.
(a) I only
(b) II only
(c) I and II but not III
(d) II and III but not I
(e) I, II and III
6. Evaluate the expression: $\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}}$
(a) $\frac{1 + \sqrt{5}}{2}$ (b) 2 (c) e (d) 4 (e) ∞

7. If a , b , c and d are positive integers, compute the numerical value of

$$\log_{10} \left(\frac{2a}{b} \right) - \log_{10} \left(\frac{c}{2b} \right) + \log_{10} \left(\frac{5c}{d} \right) - \log_{10} \left(\frac{a}{5d} \right) + \log_{10} 1.$$

- (a) 0 (b) 2 (c) 3 (d) 6 (e) None of these

8. How many triangles ABC exist such that $b = 7$, $c = 9$ and $m\angle ABC = 35^\circ$? (Assume b is the length of the side opposite vertex B and c is the length of the side opposite vertex C .)

- (a) 0 (b) 1 (c) 2 (d) Infinitely many (e) Cannot be determined

9. The local coffee shop is serving coffee, cappuccino and pumpkin spice latte. On Friday, 200 people purchased at least one of these drinks. If 121 of these people purchased coffee, 103 purchased cappuccino, 75 purchased pumpkin spice latte, 52 purchased coffee and cappuccino, 19 purchased coffee and pumpkin spice latte, and 9 people purchased all three drinks, how many purchased cappuccino and pumpkin spice latte?

- (a) 28
(b) 37
(c) 141
(d) There is not enough information provided to answer this question.
(e) None of the above.

10. If $f(x) = x^{-3}$, find the value of $f(f(f(f(f(f(2))))))$.

- (a) 1 (b) $\frac{1}{729}$ (c) 729 (d) 729^{-3} (e) None of these

11. The function f is given in the following table:

x	1	2	3	4	5
$f(x)$	5	4	1	2	3

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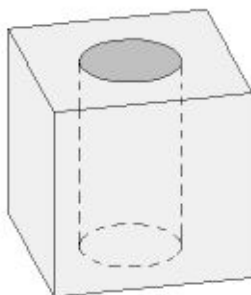
If $u_1 = 3$ and $u_{n+1} = f(u_n)$ for $n > 0$, what is the value of u_{2005} ?

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

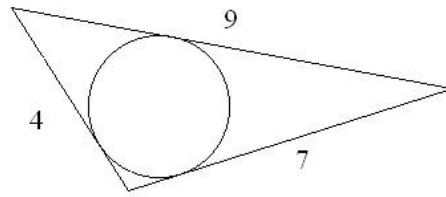
12. Adam's age equals Beth's age plus the cube root of Chris's age. Beth's age is 14 more than the sum of Chris's age and the cube root of Adam's age. Chris's age equals the cube root of Adam's age plus the square root of Beth's age. What is Adam's age? (Assume that each person's age is a whole number.)

- (a) 1 (b) 8 (c) 27 (d) 64 (e) None of these

13. Points K and L move with constant rotational velocities in a counterclockwise direction around a circle with center at the origin and radius 4. If the rotational velocities of K and L are $\frac{\pi}{2}$ and $\frac{\pi}{4}$ respectively, and if both are located at (4,0) at time $t = 0$, how many units of time will pass before the two points coincide again?
- (a) 2 (b) 4 (c) 8 (d) 12 (e) None of these
14. Suppose 60% of the people at a concert are under 24 years old. Which of the following *must* be true about the entire concert audience?
- (a) Their mean age is less than 24.
 (b) Their median age is less than 24.
 (c) Their mode age is less than 24.
 (d) More than one of the above statements must be true.
 (e) None of the above statements must be true.
15. Suppose $a^{10} = m$ and $\log_2 m = 10$. Which of the following is true?
- (a) $a^{10} = 100$ (b) $a = m$ (c) $m = 10$ (d) $a = 2^{10}$ (e) None of these
16. Sam and Jan are two of the ten students in a club. Two students from this club will be chosen at random to go on a field trip. What is the probability that Sam will be chosen to go on the field trip, but Jan will not be chosen?
- (a) $\frac{2}{45}$ (b) $\frac{8}{45}$ (c) $\frac{9}{45}$ (d) $\frac{16}{45}$ (e) $\frac{17}{45}$
17. A cylindrical hole of radius 3 is bored through a cube of side length 10, as illustrated in the diagram below. Find the surface area of the resulting solid.
- (a) $600 - 18\pi$ (b) $1000 - 90\pi$ (c) $1000 - 78\pi$ (d) $600 + 42\pi$ (e) $600 + 78\pi$



18. (First tiebreaker) Given a triangle with sides of length 4, 7 and 9 and an inscribed circle (as shown in the diagram below), what is the area of the circle?



Hint: The area of a triangle in terms of the lengths of its sides is given by Heron's formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{1}{2}(a+b+c).$$

- (a) $\frac{16\pi}{9}$ (b) $\frac{9\pi}{5}$ (c) $\sqrt{45}$ (d) $\sqrt{180}$ (e) $\frac{100\pi}{9}$
19. (Second tiebreaker) At Marvadel High School, some of the students in the marching band are in the math club, and some of the students in the math club are on the football team. However, since the marching band plays during football games, none of the students in the marching band are on the football team. Based on these assumptions, which of the following three statements *could* be true?
- I. Every student on the football team is in the math club.
II. Every student in the math club is in the marching band.
III. Some students in the math club are neither in the marching band nor on the football team.
- (a) Statements I and II, but not III
(b) Statements I and III, but not II
(c) Statements II and III, but not I
(d) All three statements could be true
(e) None of the statements could be true
20. (Third tiebreaker) What is the greatest prime number less than 900?
- (a) 891 (b) 893 (c) 897 (d) 899 (e) None of these