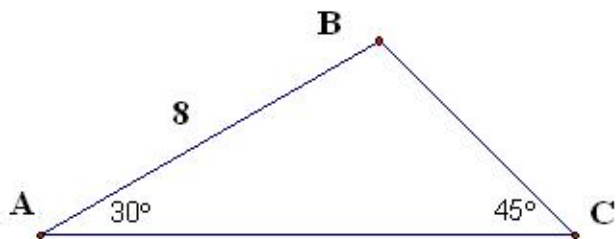


Eastern Shore High School Mathematics Competition
November 10, 2004
Individual Contest

Select the best response for each question.

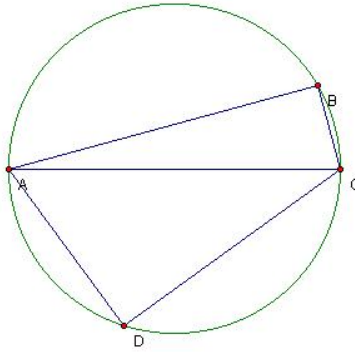
1. Find the area of the region (in the xy plane) enclosed by the lines $y = 2x$, $x = 3$ and $y = 0$.
(a) $\sqrt{18}$ (b) 6 (c) $\sqrt{45}$ (d) 9 (e) 18
2. A group of students consists of 15 males and 25 females. Among them are 10 sophomores, 18 juniors, and 12 seniors. The sophomores are comprised of 6 females and 4 males. If one student is randomly selected to receive tickets to a concert, what is the probability that the selected student is a female sophomore?
(a) $6/40$ (b) $6/25$ (c) $10/40$ (d) $6/10$ (e) $25/40$
3. What is the 2004th term of the sequence $\{a + k, a + 2k, a + 3k, a + 4k, \dots\}$ if the third term is 4 and the fourth term is 6?
(a) 2004 (b) 2006 (c) 4008 (d) 4010 (e) None of these
4. How many factors does 2004 have?
(a) 2 (b) 6 (c) 9 (d) 12 (e) 16
5. Let $f(x) = \log_{10} x$, and let $a = (10,000,000,000)^{1000}$. What is the value of $f(f(a))$?
(a) 4 (b) 400 (c) $1000a$ (d) a^{1000} (e) None of these
6. Suppose you are the payroll manager for a firm that employs 100 people. From data on the first week of July last year, the median wage was \$480, the mean wage was \$500, and the modal wage was \$475. Assuming positions and wages have remained stable, except that the overall wage level has increased 5%, how much money is needed to pay these 100 employees this year for the first week of July?
(a) \$47,500 (b) \$48,000 (c) \$50,000 (d) \$50,400 (e) \$52,500
7. If $0^\circ < A < 90^\circ$ and $\tan A = \frac{2}{3}$, then what is the value of $\sin A$?
(a) $\frac{2}{5}$ (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{13}}$ (d) $\frac{3}{\sqrt{13}}$ (e) $\frac{2}{\sqrt{5}}$

8. In the triangle shown below, $m\angle CAB = 30^\circ$, $m\angle BCA = 45^\circ$, and side \overline{AB} has length 8. What is the length of side \overline{BC} ?



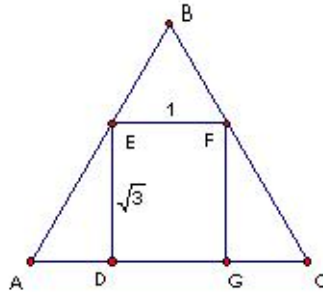
- (a) 4 (b) $\frac{8}{\sqrt{3}}$ (c) $4\sqrt{2}$ (d) $4\sqrt{3}$ (e) $5\sqrt{2}$
9. In the xy plane, what is the reflection of the point $(-1,2)$ over the line $y = x$?
- (a) $(-2,1)$ (b) $(-1,-2)$ (c) $(1,-2)$ (d) $(1,2)$ (e) $(2,-1)$
10. On planet Zelda, it is a well known fact that if one plays gibner, then one is skilled at nevil. Which of the following statements would follow logically from this?
- (a) Anyone skilled at nevil must play gibner
(b) Those who do not play gibner are not skilled at nevil
(c) Anyone unskilled at nevil must not play gibner
(d) All of the above
(e) None of the above
11. A group of students completed a survey about their favorite campus sports to watch. The sports include football, soccer, and basketball. Students were allowed to choose more than one sport on the survey. The results were as follows: 30 students said they like to watch only football; 35 said they like to watch only soccer; 100 said they like to watch only basketball; 15 like to watch football and soccer; 15 like to watch football and basketball; 20 like to watch soccer and basketball; and 5 students answered that they like to watch all three sports. How many students completed the survey form?
- (a) 110 (b) 120 (c) 155 (d) 165 (e) None of these
12. Solve the equation $\log_{\sqrt{2}} x - \log_{\sqrt{2}} 4 = 2$.
- (a) 0.5 (b) 2 (c) 6 (d) 8 (e) None of these

13. Given that AC is a diameter of the circle shown below, what is the sum of the measures of $\angle BAD$ and $\angle BCD$?

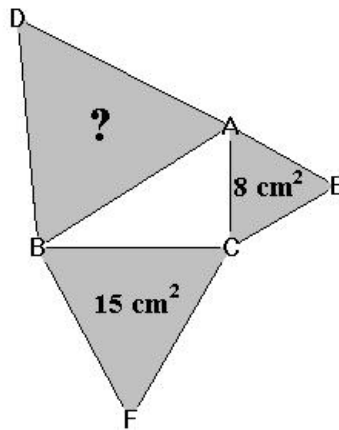


- (a) 90°
(b) 180°
(c) 200°
(d) 270°
(e) Cannot be determined from the given information
14. Points K and L move with a constant rotational velocity in a counterclockwise direction around a circle with center at the origin and a radius of 3. If the rotational velocity of K is $\frac{5\pi}{6}$ and that of L is $\frac{\pi}{3}$, and if both are located at $(3,0)$ at time $t = 0$, how many units of time will pass before K and L again coincide?
- (a) 3 (b) 3.5 (c) 4 (d) 4.5 (e) 5
15. How many real numbers k are there such that the equations $x^2 - kx + 8 = 0$ and $x^2 + k = 0$ each have two integer solutions x ?
- (a) 0 (b) 1 (c) 2 (d) 3 (e) More than 3
16. If A , B and C are sets, then the statement $x \in A \cap (B \cup C)$ is logically equivalent to which of the following?
- (a) x is in A and x is in B or C .
(b) x is in A or x is in B and C .
(c) x is in A and B but not C or x is in A and C but not B .
(d) x is in at least one of the three sets.
(e) x is in all three sets.

17. In the diagram below, $\triangle ABC$ is an equilateral triangle, $DEFG$ is a rectangle, $DE = \sqrt{3}$ and $EF = 1$. What is the perimeter of $\triangle ABC$?



- (a) $3(\sqrt{3} + 1)$ (b) 9 (c) $3(2\sqrt{3} + 1)$ (d) 15 (e) 21
18. A ball is dropped from 10 meters above a flat surface. Each time the ball hits the surface after falling a distance h , it rebounds a distance of $\frac{3}{5}h$. Find the total distance the ball travels up and down if it is allowed to continue bouncing indefinitely.
- (a) 40 meters (b) $1000\sqrt{3}$ meters (c) $1000\sqrt{5}$ meters
 (d) An infinite distance (e) None of these
19. Given that $\triangle ABC$ is a right triangle, triangles $\triangle ABD$, $\triangle ACE$ and $\triangle BCF$ are all equilateral and the areas of triangles $\triangle ACE$ and $\triangle BCF$ are 8 cm^2 and 15 cm^2 respectively, what is the area of $\triangle ABD$?



- (a) $\sqrt{120} \text{ cm}^2$ (b) 17 cm^2 (c) 20 cm^2 (d) 23 cm^2 (e) None of these
20. Suppose a city legislature consists of 12 Whigs and 8 Tories. From this group, the city's mayor wishes to select a committee consisting of 3 Whigs and 2 Tories. In how many different ways could the mayor select this committee?
- (a) $\frac{12! 8!}{9! 6! 3! 2!}$ (b) $\frac{15!}{12! 8! 5!}$ (c) $\frac{20! 3! 2!}{12! 8! 5!}$ (d) $\frac{20! 5!}{10! 5! 3! 2!}$ (e) $\frac{25!}{12! 8! 3! 2!}$