Eastern Shore High School Mathematics Competition
November 13, 2002
Team Round

Instructions: Answer as many questions as possible in the time provided. To receive full credit for a solution, show all work *and* provide a clearly written explanation. Solutions will be judged based on correctness, completeness and clarity. (Little credit, if any, will be given for a solution consisting of just a number or a single sentence.)

1. The students in a geometry class are investigating the question of whether three given line segments will form a triangle. They know that a triangle is formed if and only if the sum of the lengths of any two segments is greater than the length of the third segment.

   The students have two wooden rods, one of which is longer than the other. They cut the longer rod at a randomly chosen spot, so that they now have three rods. What is the probability that the three lengths form a triangle...

   (a) If the longer rod (before being cut) was twice as long as the shorter rod?

   (b) If the longer rod was three times as long as the shorter rod?

   (c) If the longer rod was N times as long as the shorter rod?

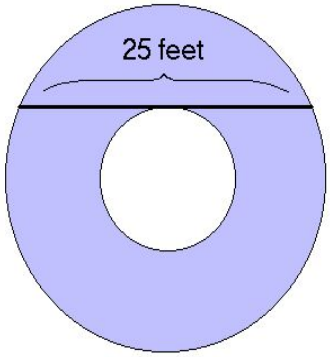
2. The sum of the ages of Mike and Brenda's children is 17. (Here we consider each child's age as a *whole number*.) Given this information, what is the largest possible product of the childrens' ages...

   (a) if Mike and Brenda have exactly two children?

   (b) if Mike and Brenda have more than two children?

3. A square is drawn on a (rectangular) coordinate plane with vertices P(1,0), Q(2,0), R(2,1) and S(1,1). A line is drawn from the origin that cuts the square into two sections, the ratio of whose areas is 2:1. What's the equation of the "cutting" line?

4. A father and son were hired to paint the floor of a merry-go-round. The floor had the shape of a washer (see the diagram below). Because the machinery to drive the merry-go-round was located in the center of the concentric circles, the son stretched a 25' measuring tape across the interior of the outer circle, but avoided the center; it lay, by chance, so that it just touched the edge of the inner circle (as shown in the diagram.) To his father's surprise, the son was able to determine the area to be painted from this measurement alone. What was the area to be painted?



5. In the following regular pentagon with sides of length 1 (for example, $AB = 1$), what is the value of $\dfrac{HI}{HJ}$, where AB, HI and HJ are the lengths of the line segments $\overline{AB}$, $\overline{HI}$ and $\overline{HJ}$ respectively?