

Eastern Shore High School Mathematics Competition  
November 13, 2002  
Individual Round

1. An author published a book every two years. When the seventh one was published, the sum of the years of publication was 13,804. In what year was the author's first book published?  
(a) 1960    (b) 1964    (c) 1966    (d) 1970    (e) 2002
2. Suppose  $x$  and  $y$  are positive numbers such that  $\log_2(x + y) = \log_2 y + 10$ . Which of the following is true?  
(a)  $x = 1023y$     (b)  $\log_2 x = 10$     (c)  $x = 2^{10}$     (d)  $x + y = 2^y + 10$     (e) None of these
3. A second number is added to a first number, and it is observed that the sum of the two numbers is equal to their product. The second number is then multiplied by one less than the first number. What can we conclude about this new product?  
(a) The product cannot be an integer.  
(b) The product must be smaller than the second number.  
(c) The product cannot be negative.  
(d) The product must be equal to the first number.  
(e) The product cannot be zero.
4. Bob's neighbor, Mr. Smith, is taking a month's vacation in June. Bob will take care of Mr. Smith's house and lawn for all of the 30 days Mr. Smith will be away. Mr. Smith has developed an unusual pay scheme to reward Bob for his month's work: Bob will receive \$10 on June 1st, and thereafter his pay for any given day will be three dollars more than it had been for the day before. How much money will Bob receive for all 30 days?  
(a) \$100    (b) \$1,605    (c) \$3,210    (d) \$10,000    (e) Bob will earn millions of dollars.
5. Which of the following expressions is equivalent to  $(x + y)^{-1}(x^{-1} + y^{-1})$ ?  
(a)  $x^{-2} + 2x^{-1}y^{-1} + y^{-2}$   
(b)  $x^{-2} + 2^{-1}x^{-1}y^{-1} + y^{-2}$   
(c)  $\frac{1}{x^{-1}y^{-1}}$   
(d)  $x^{-2} + y^{-2}$   
(e)  $x^{-1}y^{-1}$

6. Suppose  $x$ ,  $y$  and  $z$  are positive integers such that  $x^2 + y^2 = z^2$ . Which of the following must be true?

I.  $(x + y + z)^2 = 2(z + x)(z + y)$

II.  $(x - y - z)^2 = 2(z - x)(z - y)$

III.  $(x - y + z)^2 = 2(z + x)(z - y)$

- (a) I only      (b) I & III only      (c) II & III only      (d) II only      (e) I, II & III

7. Suppose  $A$  and  $B$  are events from the same sample space and  $P(A) = \frac{3}{5}$ . Under what circumstances, if any, can we conclude that the probability that  $A$  or  $B$  (or both) occurs is  $\frac{3}{7}$ ?

(a)  $A$  and  $B$  are mutually exclusive, and  $P(B) = \frac{6}{35}$ .

(b)  $A$  and  $B$  are independent, and  $P(B) = \frac{6}{35}$ .

(c)  $A$  and  $B$  are mutually exclusive, and  $P(B) = \frac{9}{35}$ .

(d)  $A$  and  $B$  are independent, and  $P(B) = \frac{36}{35}$ .

(e) Under *no* circumstances.

8. How many positive integers are factors of 2002?

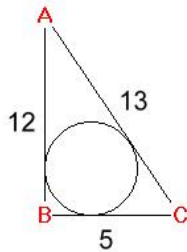
- (a) 2      (b) 4      (c) 8      (d) 16      (e) 32

9. How many prime numbers are there between 1 and 100?

- (a) 9      (b) 16      (c) 25      (d) 49      (e) 50

10. A circle is inscribed inside a 5-12-13 triangle, as shown below. What is the area of the circle?

- (a)  $2\pi$       (b) 10      (c)  $4\pi$       (d)  $5\pi$       (e) 20



11. Which of the following is the largest?

- (a)  $2^{10,000,000,000}$
- (b)  $3^{6,000,000,000}$
- (c)  $5^{4,000,000,000}$
- (d)  $6^{3,500,000,000}$
- (e)  $10^{1,000,000,000}$

12. If  $(f \circ g)(x) = x^4$  and  $f(x) = (x + 1)^2$ , then a correct choice for  $g(x)$  is:

- (a)  $g(x) = x^2 - 1$
- (b)  $g(x) = x^2 + 1$
- (c)  $g(x) = x^4 + 1$
- (d)  $g(x) = (x + 1)^4$
- (e)  $g(x) = \frac{x^4}{(x + 1)^2}$

13. On the  $xy$ -plane, for fixed  $a$ , the symmetric axis of the curve  $2x^2 + y - 4ax - \frac{3}{2} = 0$  is

- (a)  $x = a$     (b)  $x = -a$     (c)  $x = -2a$     (d)  $x = 2a$     (e) None of these

14. What is the coefficient of  $x^4y^6$  in the expansion of  $(2x + 3y)^{10}$ ?

- (a)  $\frac{10!}{4! \cdot 6!}$     (b)  $2^4 \cdot 3^6 \cdot \frac{10!}{4! \cdot 6!}$     (c)  $\frac{4! \cdot 6!}{2! \cdot 3!}$     (d)  $2^6 \cdot 3^4 \cdot \frac{10!}{4! \cdot 6!}$     (e)  $2^4 \cdot 3^6 \cdot \frac{4! \cdot 6!}{10!}$

15. If  $A$  and  $B$  are sets such that  $(A \cup B) \subseteq (A \cap B)$ , then which of the following, if any, must be true?

- (a)  $A = B'$     (b)  $A = \emptyset$     (c)  $B = \emptyset$     (d)  $A = B$     (e) None of these

16. The arithmetic mean (average) of the heights of the players on this year's basketball team was 72 inches. However, the tallest player, who is 75 inches tall, had to leave the team due to an injury. As a result, the arithmetic mean of the players' heights is now 71.8 inches. How many players were originally on the team?
- (a) 10    (b) 15    (c) 16    (d) 20    (e) Cannot be determined
17. The graphs of  $y = -|x - a| + b$  and  $y = |x - c| + d$  intersect at the points  $(2, 5)$  and  $(8, 3)$ . What is the value of  $a + c$ ?
- (a) 7    (b) 8    (c) 9    (d) 10    (e) 12
18. Assume each of the following premises is true:
- All tennis players wear funny clothes.
  - No one who wears funny clothes subscribes to "Southern Living."
  - Some golfers subscribe to "Southern Living."
  - John plays tennis.

Based on these premises, which of the following *must* be true?

- (a) John does not play golf.  
(b) John does not subscribe to "Southern Living."  
(c) Some golfers play tennis.  
(d) Some tennis players play golf.  
(e) No one who wears funny clothes plays golf.
19. Increase a rectangle's width by 1 and decrease its length by 1, and its area is decreased by 4. But decrease its length by 1 and increase its width by 2, and its area is increased by 16. What is the area of the rectangle?
- (a) 16    (b) 45    (c) 88    (d) 180    (e) 504
20. Suppose that there were six students in a class, three of whom were girls. Two students were selected from the class at random. You are told that at least one of the selected students was a girl. Given this information, what is the probability that *both* of the selected students were girls?
- (a)  $1/40$     (b)  $1/4$     (c)  $1/3$     (d)  $19/39$     (e)  $1/2$