

Eastern Shore Mathematics Competition - November 14, 2001
Individual Problems

(Correct answers are highlighted in boldface.)

1. The sum of the reciprocals of the roots of $x^2 + ax + b$ is

(a) $-\frac{a}{b}$ (b) $-\frac{b}{a}$ (c) $\frac{a}{b}$ (d) $\frac{b}{a}$ (e) None of these

2. In a recent charity event, Ellen collected \$2 per mile for a total of \$52. To do so, she walked for a certain time at 3 miles per hour (mph), and the rest at 4 mph. If she had walked 4 mph for the same time that she actually walked 3 mph, and vice versa, she would have collected \$60. How long did her walk take?

(a) 3 hours (b) 4 hours (c) 6 hours (d) 12 hours (e) **None of these**

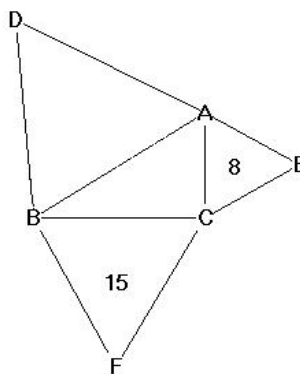
3. Consider the series $r^2 + r^4 + r^6 + r^8 + r^{10} + \dots$. The third term is 16 times the fifth term. The sum of this series is:

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{4}{3}$ (e) 2

4. A list of six numbers has a mean average of 75. When the lowest number is replaced by 55, the new mean is 80. What is the difference between the new lowest number of 55 and the original lowest number?

(a) 5 (b) 10 (c) 25 (d) **30** (e) Cannot be determined

5. Given that $\triangle ABC$ is a right triangle, triangles $\triangle ABD$, $\triangle ACE$ and $\triangle BCF$ are all equilateral and the areas of triangles $\triangle ACE$ and $\triangle BCF$ are 8 and 15 respectively. What is the area of $\triangle ABD$?



(a) **17**

(b) 20

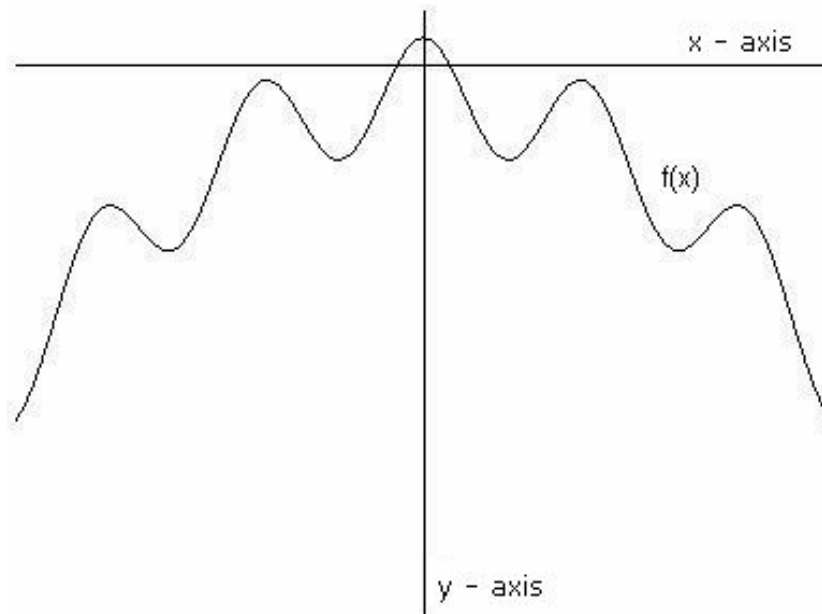
(c) 23

- (d) None of the above but can be determined.
 (e) Cannot be determined from the given information.

6. To fill an open position, a company must choose one of three candidates: Bob, Barbara or Mike. Bob and Mike are equally likely to be hired; however, due to her superior qualifications for the job, Barbara's chance of being hired is 20% higher than Bob's chance of being hired. What is the probability that Barbara will be hired?

- (a) 0.0625 (b) 0.20 (c) 0.3125 **(d) 0.375** (e) 0.5

7. The function $f(x)$ is defined by the graph shown below. How many solutions are there to the equation $f(x) = x^2$?



- (a) 0 (b) 1 **(c) 2** (d) 3 (e) 4

8. Suppose that A is a set and that every element of A is an integer. Which of the following statements must be true?

- I. If x is an integer, then x is an element of A .
- II. If x is not an element of A , then x is not an integer.
- III. If x is not an integer, then x is not an element of A .

- (a) I and II (b) II and III (c) I and III (d) I only **(e) III only**

9. Twelve children show up for a costume party. Nine are wearing Disney costumes. Eight are wearing masks. Each is wearing a Disney costume, a mask, or both. How many are wearing both a Disney costume and a mask?

- (a) 1 (b) 3 **(c) 5** (d) 7 (e) Cannot be determined

10. How many different rearrangements are there of the word ANNUAL?

- (a) 36 (b) 64 (c) 120 **(d) 180** (e) 720

11. Assume that each of the following statements is true:

All refrigerators use electricity.

Not all appliances require electricity.

You cannot use things that require electricity while camping in a national forest.

Which of the following may be concluded from these statements?

I. Refrigerators are not appliances.

II. You cannot use appliances while camping in a national forest.

III. You cannot use refrigerators while camping in a national forest.

IV. Not all appliances are refrigerators.

- (a) I and IV (b) II and III **(c) III and IV** (d) II, III and IV

(e) I, II, III and IV

12. Inscribe an equilateral triangle inside a circle of radius R , and then inscribe a circle inside the triangle. Let the radius of the smaller circle be r . What is $\frac{R}{r}$?

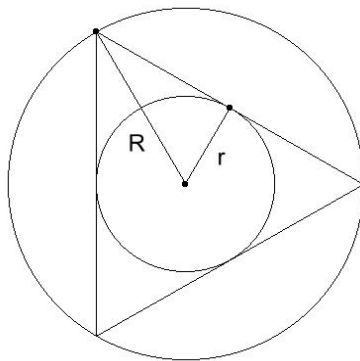
(a) $\frac{\sqrt{3}}{2}$

(b) $\frac{\pi}{2}$

(c) $\frac{1+\sqrt{5}}{2}$

(d) None of the above but can be determined. (*Actual answer: 2*)

(e) Cannot be determined.



13. The symbol $[x]$ denotes the greatest integer which is less than or equal to x . Suppose $\frac{x[x]}{|x|^2} = 1$.

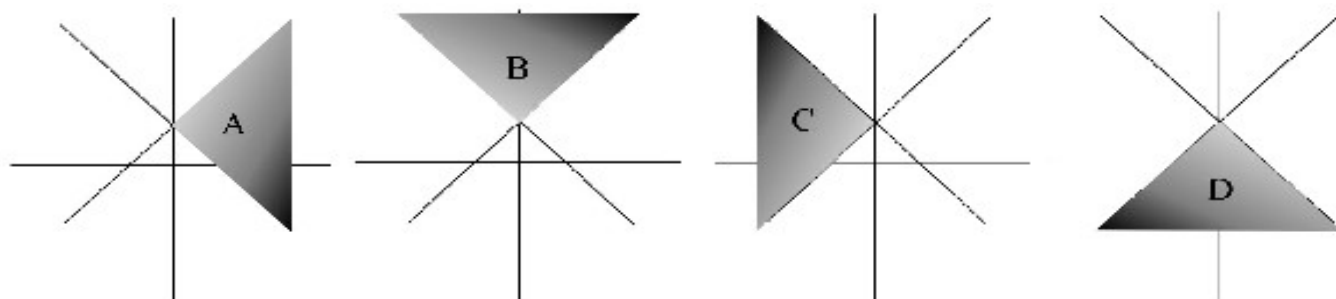
What may we conclude about x ?

(a) x must equal 1

(b) x must be positive

- (c) x can be any integer
 (d) x must not be an integer
 (e) **None of these** (x can be any integer except zero.).

14. In which region do the points satisfy both $y - x > 1$ and $y < 1 - x$?



- (a) A (b) B **(c) C** (d) D (e) More than one of these regions

15. Suppose $\log_a 2 = m$, $\log_a 3 = r$, $\log_a 5 = s$ and $\log_a 11 = t$. What is $\log_a 990$?

- (a) $2mrst$ **(b) $m+2r+s+t$** (c) $m+r+s+t$ (d) $2+9\cdot 9$ (e) None of these

16. A drawer contains 12 black socks and 4 blue socks. Two socks are chosen from the drawer at random. What is the probability that they match?

- (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ **(c) $\frac{3}{5}$** (d) $\frac{3}{4}$ (e) None of these

17. Harold and Horace were gifted gamblers. On one particular day, each managed to double his money every hour! Harold began the day with \$20, and gambled for 20 hours. Horace began the day with \$40, and finished with the same amount of money as Harold. For how many hours did Horace gamble that day?

- (a) 10 **(b) 19** (c) 21 (d) 40 (e) None of these

18. How many positive integers less than 10,000 are there whose digits sum to 4?

- (a) 15 (b) 25 **(c) 35** (d) 45 (e) None of these

(Those 35 integers are: 4, 40, 400, 4000, 31, 310, 3100, 301, 3010, 3001, 13, 130, 1300, 103, 1030, 1003, 22, 202, 2002, 220, 2020, 2200, 2110, 2101, 2011, 1201, 1210, 211, 1021, 1120, 121, 112, 1012, 1102, and 1111.)

19. Evaluate: $1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}$.

- (a) $\sqrt{2}$** (b) 2 (c) π (d) ∞ (e) None of these

(Hint: Let x be the value of the continued fraction. Then notice that x must be a root of the equation $x = 1 + \frac{1}{1+x}$.)

20. Three smaller, identical circles are inscribed inside a larger circle of radius r , as shown in the diagram below. What is the combined area of the three smaller circles?

(a) $\left(\pi - \frac{\sqrt{3}}{2}\right)r^2$

(b) $\pi r^2 - 3\left(\frac{\sqrt{2}}{2} - 1\right)^2 \pi$

(c) $\left(63 - 36\sqrt{3}\right)\pi r^2$

(d) None of the above but can be determined.

(e) Cannot be determined.

