

# The Thirty-Eighth Annual Eastern Shore High School Mathematics Competition

November 16, 2022

## Individual Contest Exam

### Instructions

There are twenty problems on this exam. Select the best answer for each problem.

Your score will be the number of *correct* answers that you select.

**There is no penalty for incorrect answers.**

The use of a calculator is **not** permitted on this exam.

No computational work is required for any of your multiple choice responses.

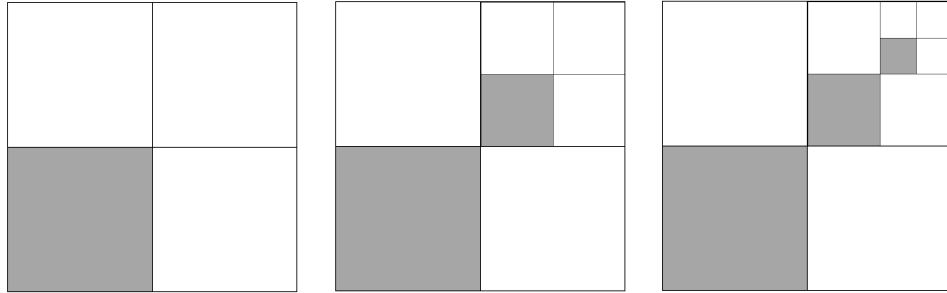
However, in the event of tie scores, after the multiple choice responses have been checked for problems 1-20, the responses and/or written computational work on the enclosed form for problems #18, #19 and #20 will then be used as tiebreakers.

1. Assuming  $x > y$ , calculate the ratio  $\frac{x}{y}$  if:

$$2 \log_{2022}(x - 4y) = \log_{2022}(3x) + \log_{2022}(3y)$$

- a. 8      b. 1      c. 3      d. 4      e. 16

2. A unit square is shaded according to a multi-step process. The first three steps (step 1, step 2, and step 3) are shown in the figure below.

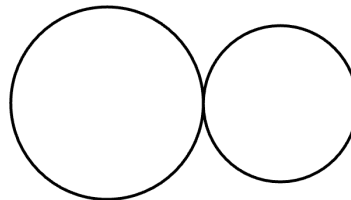


What fraction of the total area of the original unit square will be shaded after step 6 is completed?

- a.  $\frac{1}{4}$       b.  $\frac{341}{1024}$       c.  $\frac{1365}{4096}$       d.  $\frac{1}{3}$       e.  $\frac{4095}{4096}$

3. Kevin and John are solving a problem on a High School Math Exam. Kevin's chance to solve the problem correctly is 60% while John's chance of a correct solution is 30%. What is the probability that only one of them will solve the problem correctly?
- a. 0.12      b. 0.18      c. 0.42      d. 0.54      e. 0.9

4. Two discs touch (as shown) and rotate together without slipping. The larger disc has a diameter of 40 cm and rotates 12 times per minute. What is the diameter,  $d$ , of the smaller disc if it rotates once per second? Note: the discs are not drawn to scale.



- a.  $d = 2$  cm      b.  $d = 4$  cm      c.  $d = 6$  cm      d.  $d = 8$  cm      e.  $d = 12$  cm
5. A fair, well-balanced die, is rolled twice. What is the probability that the first roll results in a number greater than 3 while the second roll results in a number less than 3?
- a. 0      b.  $\frac{1}{2}$       c.  $\frac{1}{3}$       d.  $\frac{1}{6}$       e.  $\frac{1}{36}$
6. Consider sets A, B, and C in the universal set  $\mathbb{R}$  to be defined as:

$$A = \{-1, 0, 1, 2, 3\} \quad B = [0, 2] \quad \text{and} \quad C = (1, 3)$$

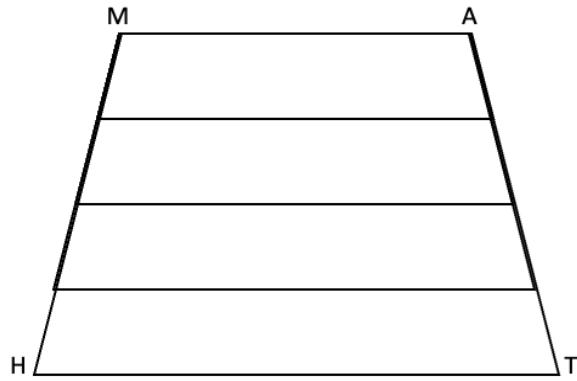
What set would represent  $B \cup C$ ?

- a.  $[0, 3]$       b.  $[0, 3]$       c.  $\{x \in \mathbb{R} \mid 0 \leq x < 3\}$       d.  $(1, 2]$       e. a and c are both correct

7. If  $\frac{3x}{x-1} - \frac{2x}{x+2} = \frac{3x-6}{(x-1)(x+2)}$ , what interval contains the solution(s)?
- a.  $-5 < x < -2$       b.  $-2 < x < 0$       c.  $-3 < x < -1$       d.  $0 < x < 3$       e.  $2 < x < 5$
8. Which of the following statements are true: (for all of these assume  $x$  is an integer)
- I If  $x < 3$  then  $x + 1 < 4$ .  
 II If  $3 > 4$  then  $\pi$  is an integer.  
 III If  $5 > 7$  then  $\pi$  is irrational.  
 IV If  $x$  is divisible by 7 then so is  $2x$ .
- a. I and IV      b. I,III and IV      c. I,II, and IV      d. I      e. all of these
9. What is the units digit of the expression  $3^{2022} - 2^{2022}$ ?
- a. 6      b. 1      c. 3      d. 5      e. 4
10. A survey of a group's viewing habits over the last year revealed the following information:
1. 25% watched football
  2. 27% watched baseball
  3. 28% watched soccer
  4. 11% watched football and baseball
  5. 13% watched baseball and soccer
  6. 9% watched football and soccer
  7. 5% watched all three sports.
- Calculate the percentage of the group that watched none of the three sports during the last year.
- a. 24%      b. 36%      c. 48%      d. 52%      e. 95%
11. Consider the statement: Dilip is taking Calculus or Discrete Mathematics. This will be a false statement if and only if which of the following is true?
- I Dilip is not taking Calculus and Dilip is not taking Discrete Mathematics.  
 II Dilip is not taking both Calculus and Discrete Mathematics.  
 III Dilip is either not taking Discrete Mathematics or not taking Calculus.  
 IV Dilip may be taking both Calculus and Discrete Mathematics.
- a. I and II      b. II and III      c. IV      d. I      e. none of these
12. Simplify the expression  $(\sin(\alpha) \cos(\beta) - \cos(\alpha) \sin(\beta))^2 + (\cos(\alpha) \cos(\beta) + \sin(\alpha) \sin(\beta))^2$ :
- a.  $-1$   
 b.  $0$   
 c.  $1$   
 d.  $2$   
 e. None of the above

13. Find the inverse of  $f(x) = \frac{3x-2-4}{5}$
- $f^{-1}(x) = \frac{6(5y+4)+2}{3}$
  - $f^{-1}(x) = \frac{6(5x-4)+2}{3}$
  - $f^{-1}(x) = \frac{6(5x+4)-2}{3}$
  - $f^{-1}(x) = \frac{6(5x+4)+2}{3}$
  - $f^{-1}(x) = \frac{6(5x+4)+2}{30}$
14. Note that the number 2022 is divisible by both 2 and 3. What is the smallest number after 2022 consisting of only 2's and 0's that is divisible by both 2 and 3?
- 20022
  - 2202
  - 2220
  - 2222
  - 20202
15. The minute and hour hand of an analog clock coincide approximately every 65 minutes. How many times do they coincide during a single day?
- 20
  - 22
  - 19
  - 21
  - 23
16. How much of a 20% acidic solution should be added to 5 milliliters of a 75% acidic solution to form a 30% acidic solution?
- 22.5 milliliters
  - 125 milliliters
  - 0.225 milliliters
  - 25 milliliters
  - 0.125 milliliters
17. The sum of 4 consecutive integers is 2022. What is the sum of the digits in the third integer?
- 12
  - 10
  - 13
  - 11
  - 9
18. Consider the following sequence:
- $$1, 4, 7, 10, 13, \dots$$
- What is the smallest value of  $n$  for which the sum of the first  $n$  terms of the sequence is greater than 1000?
- $n = 26$
  - $n = 28$
  - $n = 34$
  - $n = 35$
  - $n = 52$
19. Let  $f(x) = \frac{ax+b}{cx+d}$ , such that  $f(x)$  is not the identity function, and suppose  $f \circ f(x) = x$ . Which of the following statements are true?
- $a + d = 0$
  - $b + c = 0$
  - $ad = bc$
  - All of the above are true.

20. Quadrilateral MATH is an isosceles trapezoid. The five horizontal segments are parallel and equally spaced. The length of segment MA is  $3\frac{1}{3}$  units and the length of segment HT is  $5\frac{2}{3}$  units. Let  $S$  be the sum of the lengths of the five parallel segments. Which of the following statements is true?



- a.  $22.4 < S < 22.5$       b.  $22.5 \leq S < 22.6$       c.  $22.6 < S < 22.7$       d.  $22.7 < S < 22.8$       e.  $22.8 < S < 22.9$