Children's Developing Strategies for Dealing with Problems that Require Multiplicative Reasoning

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Introduction

It is imperative that students develop conceptual understanding of multiplication and division to promote their mathematical development (National Council of Teachers of Mathematics, 2014). Such conceptual understanding includes being able to navigate among multiple representations of multiplication and division (Common Core State Standards Initiative, 2010). However, research demonstrates that students generally display weak conceptual understanding of multiplication and division, relying heavily on algorithmic proficiency instead (Dubé & Robinson, 2018).

Our study involved creating lessons for a group of students entering fifth grade. The lessons focused on conceptual understanding and multiple representations of multiplication and division.

The purpose of this study was to design a problem sequence to develop students' abilities to select strategies that are appropriate and efficient for solving problems requiring multiplicative reasoning.

References:
Common Core State Standards Initiative. (2010). *Common core state standards for mathematics*. Retrieved from http://www.corestandards.org/math
Dubé, A. K., & Robinson, K. M. (2018). Children's understanding of multiplication and division: Insights from a pooled analysis of seven studies conducted across seven years. *British Journal of Developmental Psychology, 36*, 206–219.
National Council of Teachers of Mathematics (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

Literature Review

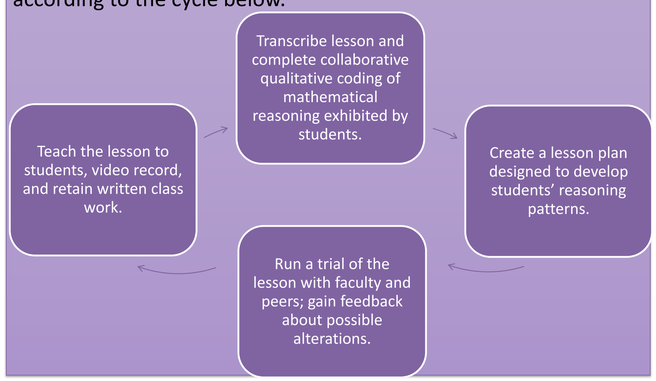
The Common Core State Standards that provided concrete goals for our instructional sequence were as follows:
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Teaching Strategies from the Literature
To promote our goal of increasing our students' conceptual understanding of multiplication and moving them into division, we created opportunities for students to create their own engaging multiplication and division word problems (Essex & McCormick, 2017). Our lessons emphasized the idea of equal-sized groups and their importance in multiplication and division (McDuffie & Sullivan, 2009). Our instructional sequence encouraged students to move from working with physical manipulatives to using their conceptual understanding to work with larger, more abstract multiplication and division problems (Gurganus & Wallace, 2005).

References:
Essex, N.K., & McCormick, K.K. (2017). Capturing children's multiplication and division stories. *Teaching Children Mathematics, 24* (1), 40-47.
Gurganus, S. P, & Wallace, A. H. (2005). Teaching for mastery of multiplication. *Teaching Children Mathematics, 12*(1), 26-33.
McDuffie, A. R., & Sullivan, A. D. (2009). Connecting multiplication contexts and language. *Teaching Children Mathematics, 15*(8), 502-511.

Methodology

Participants: Our cohort consisted of 2 girls and 2 boys who just completed fourth grade.
They were assigned the pseudonyms Jessie, Chase, Robbie, and Sarah.
Attendance: Jessie, Chase, and Robbie attended 9/9 sessions. Sarah attended 8/9 sessions.
Instructional Cycle:
Over the course of nine weeks, we conducted individual 30-minute pre-interviews with the students, taught 7 one-hour lessons, and concluded with individual 30-minute post-interviews that mirrored the pre-interview script. Each lesson was tailored towards the students' needs, which were documented and analyzed each week according to the cycle below.



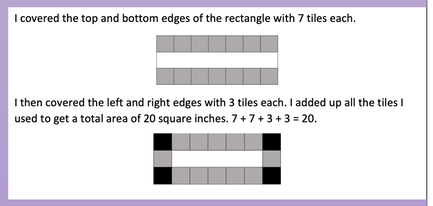
Methodology

INTERVIEW TASKS

KEY TASK 1: A photographer has 191 photos of animals and 234 photos of plants. He wants to put all of the photos into photo books. Each page of the photo books holds 8 photos. What is the fewest number of pages he could use in the photo books?

KEY TASK 2: During a class trip to an apple farm, a group of students picked 2,436 apples. They packed them into 6 boxes to take to the local food bank. If each box held the same number of apples, how many apples were in each box?

KEY TASK 3:
Donald uses square tiles measuring 1 in. on each side to find the area of a rectangle. His reasoning is shown here:

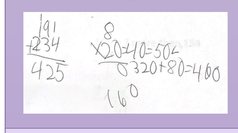


A. Could you act out and explain what Donald did with his tiles using these tiles (point out the physical tiles again)?
B. What multiplication sentence could you use to find the area?
C. Could you use these tiles to find the area of the following rectangle?

Empirical Teaching and Learning Trajectory:

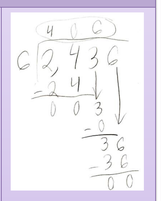
Initial Assessment Results

Key Task 1: Multi-Step Problem; Multiple Operations
The primary operation was division in this problem. Sarah chose to multiply, Robbie chose to subtract, and Jessie chose to add. Chase was able to solve the problem using an alternative algorithm. Chase's work is shown below.



In his work, Chase utilized multiplication to approach division.

Key Task 2: Division Word Problem
Sarah and Chase were able to complete the problem correctly. Again, Chase utilized a "combine groups and add" approach. Sarah's work is shown to the side. Robbie and Jessie were unable to choose the correct operation.



Key Task 3: Understanding Area
Sarah and Chase were able to recreate the diagram using their tiles and correctly discuss area in the context of rectangular arrays. Jessie and Robbie misinterpreted the diagram and struggled to explain area using the tiles. They were unable to communicate that the area must cover the entire figure.

Lessons 1-2: Word Problem Interpretation

Lesson One: Our first lesson focused on developing word problem comprehension in order to help the students understand which operations to choose in different contexts. We gave each student an individualized word problem to work on their own, and then allowed them to work in pairs. We noted that students had trouble differentiating between the number of groups and group size, especially with more abstract manipulatives like snap cubes. Some students created their own drawings to help them understand the problem.

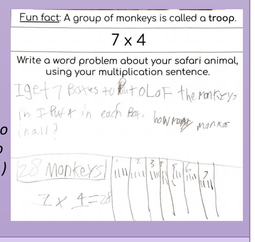
Jessie's Work

Lesson Two: For lesson 2, students were given the opportunity to create division and multiplication word problems for a partner with specific parameters such as: *product cannot exceed 275, word problem must include the fraction ¼.* They were asked to model their problems using pictures ***or*** manipulatives. Afterwards, students shared their word problems with the group. Students struggled with correctly using words that usually indicate multiplication and division. When students found it difficult to model using manipulatives, we found that artistic pictures helped them contextualize their problems.

Lessons 3-4: Equal-Sized Groups

Lesson Three: Students focused on the role of equal-sized groups in multiplication by looking at collective nouns for animal groups such as dazzle and pride. Jessie, Sarah, and Robbie had difficulty recognizing the difference between the number of groups and the within group count.

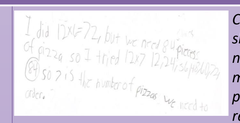
Robbie's word problem and mathematically detailed visual representation. (Students were also given the option to use manipulatives.)



Lesson Four: To draw the students away from relying on keywords, we shared a variety of tasks that engaged the students in the different phases of solving word problems. Robbie and Jessie had difficulty focusing on the tasks without physical manipulatives and drawings, while Chase was able to correctly interpret the various tasks. Sarah was absent.

Lessons 6-7: Scaling Up

Lesson Five: To help contextualize multiplication and division, children were given a multi-step word problem. *We want to have a pizza party for the 28 people in our math program. We found that a large pizza cut into 12 pieces would be our cheapest option. If we order 3 pieces for each person, how many pizzas do we need to order?*

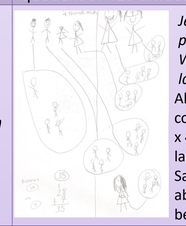


Chase's work shows his ability to navigate through a multi-step word problem and reflect on his work.

All students realized they would need to order 420 pieces of pizza and used a multiplicative strategy (similar to the one above) to find the quotient of 420 divided by 12.

Lesson Six: After the success of the prior week, an additional layer of complexity was added to the problem.

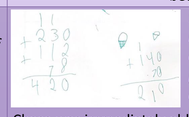
Dr. Bergner wants to invite more people to the party. She said that each of the 28 people in the program will bring 4 guests. How many people are there now? How many pizzas do we need to order?



Jessie's drawing of 7 people with 4 guests each. We extended this to the larger problem.

All students initially considered the product 28 x 4 and found working with larger numbers a challenge. Sarah thought critically about the difference between 28 x 5 and 28 x 4.

Lesson Seven: *We have 78 scoops of peanut butter ice cream, 112 scoops of strawberry ice cream, and 230 scoops of vanilla ice cream. Half of the 140 people at the picnic brought their dogs. We want to share the ice cream scoops equally amongst all of the people and dogs at the picnic. How many scoops does each receive?*



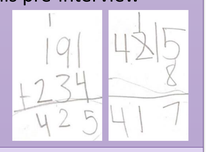
Sarah's work illustrates how she broke up the components of the word problem into a set of addition problems.

Chase was immediately able to solve and explain his solution. Sarah struggled with which quantities to add, but resolved her misunderstanding. Robbie added all of the quantities together. Jessie was instructed individually in order to solidify her understanding of the pizza problem.

Post-Assessment Results

Key Task 1: Sarah incorrectly chose to multiply initially, but reflected on her work and adjusted her solution correctly. Robbie completed the problem correctly using partial quotients. Chase's solution was very similar to his pre-interview solution.

As shown to the right, Jessie chose to add and then subtract. She still had trouble with the second part of the problem.



Key Task 2: Chase used the same strategy from the pre-interview to solve this problem. Sarah correctly used the standard algorithm for division. Robbie attempted to use partial quotients, but struggled with the larger numbers. Jessie was unable to complete the problem.

Key Task 3: Jessie and Sarah struggled to model the diagram correctly, but Sarah used colored tiles and found the area. Jessie could not find the area or model it. Chase modeled the diagram correctly and computed the area, but could not connect the product 3 x 7 to the area of 21 tiles. Robbie was unable to complete due to time.

Reflection and Discussion: Throughout the experience, our students displayed a high level of competence with standard algorithms, but lacked conceptual understanding of the operations. It was difficult to break our students' habits of focusing on key words in word problems and/or defaulting to what they claimed was an "easier", but incorrect, operation. Our students also experienced difficulty navigating between the number of groups and group size in multiplicative word problems. To discourage reliance on key words, we utilized physical manipulatives and student drawings to encourage understanding of word problems. We then scaled-up previously discussed word problems, so that students could work with larger quantities than they had previously experienced. We suggest that teachers involve students in modeling problems that rely on multiplicative reasoning and not solely focus on mastery of the standard algorithms. Such teaching strategies provide opportunities for students to develop conceptual understanding.