Supporting Online Discourse about Proportional Relationships

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Introduction

Proportional reasoning is key to the middle school mathematics curriculum (CCSSM, 2010). However, students struggle with differentiating between reasoning additively and multiplicatively (Langrall & Swafford, 2000). Also, it can be challenging for students to make sense of multiple representations that foster multiplicative reasoning (Williams-Candek, 2016).

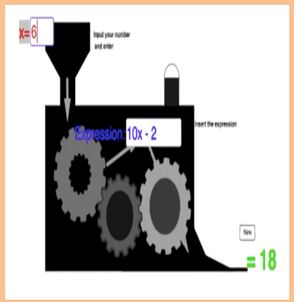
Our challenge was to help students develop proportional reasoning in an online environment. Our research questions were:

1. What technique(s) help to foster online discourse?
2. How does students' proportional reasoning develop during online discourse?

Literature Review

The *Illustrative Mathematics* (2020) curriculum requires that seventh graders, "understand ratio concepts and use ratio reasoning to solve real-world and mathematical problems". The literature contains numerous strategies to help students reach this goal. Research-based strategies we used included:

- Double number lines (Watanabe, 2015)
- Discussion Prompts (Sun et. al, 2018),
- Explicit tables (Burton, 2017)
- Function Machines (Reeves, 2006)

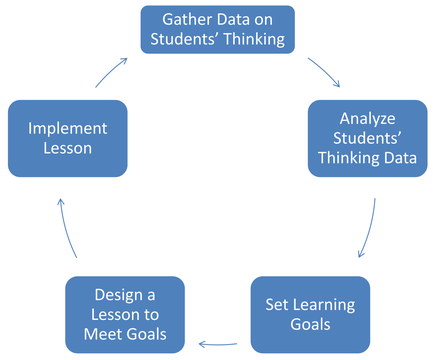


Methodology – Participants and procedure

We had 4 students who just finished 7th grade. Their pseudonyms are Steve, Olivia, Mike, and Maya. All students participated in our seven 1-hour sessions and pre- and post-interviews.

All interviews and lessons were video-recorded and transcribed via Zoom. We analyzed the lesson videos, using edTPA prompts for planning, implementation, and assessment (SCALE, 2019). Key interview tasks shown below came from the *Illustrative Mathematics* curriculum (Illustrative Mathematics, 2020).

Instructional Cycle



Key Task 1:
Carlos bought 6 pounds of bananas for $3.00

a. What is the price per pound of the bananas that Carlos bought?
b. What quantity of bananas would one dollar buy?
c. Can you share another way to solve part b?

Key Task 2:
Lin rode a bike 20 miles in 150 minutes. If she rode at a constant speed

a. How far did she ride in 15 minutes?
b. How long did it take her to ride 6 miles?
c. What was her pace in minutes per mile?
d. Ty claims that Lin would have ridden 10 miles in 75 minutes. Do you agree or disagree? Why?

References

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Sun, K., Baldinger,E., & Humphreys, C. (2018). Number talks: Gateway to sense making. *Mathematics Teacher, 112*(1) 48-54.
Watanabe, T. (2015). Visual reasoning tools in action. *Mathematics Teaching in the Middle School, 21*(3) 152-160.
Williams-Candek, M. (2016). All talk and more action. *Mathematics Teaching in the Middle School*. *22*(3) 162-170.

Initial Assessment results (Week 1)

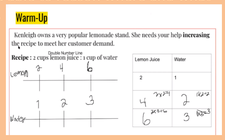
Key Task 1:
The primary focus in this problem was proportional reasoning. Steve, Mike, and Olivia divided the total cost $6 into 3 lbs. to correctly determine the price per 1 lb. Maya struggled to interpret this problem multiplicatively and added 3 + 6 to get 9 and added another 9 to get 18.

Key Task 2:
The primary focus in this problem was for students to evaluate the given cycling rate to answer the questions. Steve and Olivia showed strength in their ability to reason proportionally by stating that "every 15 minutes is 6 miles" to work through elements of Key Task 2. Mike was able to solve part D by stating, "I agree because75 minutes plus 75 minutes is 150 minutes and if she rode 20 minutes … that would be adding another 10." In part A he had the right reasoning for determining the pace, however he was not able to translate that into how many miles would have been traveled after 15 minutes. Maya, however, had trouble understanding the problem multiplicatively. In Part A, she stated "I minus 20 minus 6" to get 14 minutes, which was not the correct solution.

Empirical Teaching and Learning Trajectory:

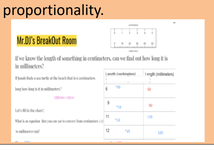
Discovering Proportional Relationships with Visual Tools

Lesson 1: Creating Proportional Recipes
First, students created equivalent ratios by doubling and tripling a lemonade recipe. Students were prompted to use the double number line to represent the recipes they had just created. Students were able to reason where and what order that ratios belonged.



Lesson 2: Dimes and Quarters
Students were asked guided questions relating to the amount of quarters and dimes. For example we asked, "how many quarters are in a dime." Students successfully reasoned about such questions but used strategies other than the double number lines we introduced in the previous lesson.

Lesson 3: Ruler Conversions
To solidify students' understanding of double number lines, we introduced the task of converting between centimeters and millimeters. Students successfully used double number lines and tables to do so. They were also able to write an equation to covert centimeters and millimeters and identify its constant of proportionality.



Representing Proportional Relationships with Equations and Tables

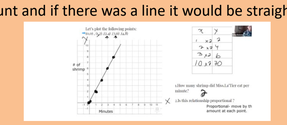
Lesson 4: Pints of Ice Cream
Students were shown a table representing the relationship between pints of ice cream and number of people. They were to fill in the missing quantities within the given table and write an equation for the relationship between the two variables. All 4 students were successful in filling in the chart and producing an equation with the assistance of instructors, when necessary.

Lesson 5: Buying Shrimp by the Pound
First, students were asked to identify which equation(s) and table represented the relationship between the total amount of shrimp and number of pounds, given that there were 16 shrimp per pound. All students correctly explain and chose the correct equations to represent the shrimp scenario. Three out of four students identified the correct table. Mike used the table to help identify the equation that represented the scenario.

Lesson 6: Let's Party Like Shrimp is 99 cents
For the opening activity students were tasked with identifying an equation that represented the discounted shrimp. Students had to represent the relationship between the total cost of shrimp and number of pounds. Initially, 3 students were able to appropriately explain and chose the correct equations to represent the shrimp scenario. The student who was unable to choose the correct equation made the connection that 50% is equivalent to 1/2. Olivia was able to explain to Steve that he needed to take 50% off $10 to find the new price and then multiply by 5 by the amount of pounds to find the total cost.

Representing Proportional Relationships with Graphs

Lesson 7: Shrimp Eating Contests & Clothing Stores
This activity gave students a list of coordinate points, students were required to first plot the points on the graph, identify the amount of shrimp eaten per minute, and determine if the relationship was proportional. All students found the amount of shrimp eaten per minute. Furthermore, Olivia described why the relationship was proportional. She said, "Yes, the graph is proportional because the points move up by the same amount and if there was a line it would be straight through the points."



Online Discourse Techniques

- ***Breakout rooms:*** This Zoom feature allowed us to pair students. It was especially helpful for Maya and Mike, who were reluctant to participate in whole-group discussions.
- ***Notice and Wonder:*** This routine allowed all students to think deeply about the context, while providing their thoughts on the content before we explored furtehr. In turn, it created a channel for students to reflect and conduct group discussions.
- ***Silent thumbs ups:*** For this technique we had students raise their thumb quietly when they agreed with what the instructor said or they had an answer. Oftentimes this technique ensured that students were responsive and actively listening during the learning segment.
- ***Prompting student-student interaction:*** This routine had students respond or paraphrase other student responses throughout the learning segment. This helped students develop reasoning and communication skills.
- ***Chat box:*** This feature allowed us to receive questions and/or answers from students via chat. This provided students with a more discreet way to communicate who had an answer, and it provided an undisturbed space for students to think.

Post-Assessment Results (Week 9)

Key Task 1:
Olivia and Steve's solution to key task 1 did not vary from the pre-assessment. However, Steve was able to give an additional solution to the problem. He provided a solution resembling reasoning done with double number lines over the summer. Steve stated, "50 cents + 50 cents which equals $1 and add 1 pound + 1 pound which equals 2 pounds." Mike demonstrated proportional reasoning in Part A even though his solution was not completely correct. He stated that, "3 pounds would be $1.50." Mike did not continue his train of thought to compute the price of one pound. Even though Maya did not solve the task correctly, she began to recognize that multiplication and division were relevant to its solution.

Key Task 2:
Olivia and Steve maintained their ability to reason proportionally and answered the question correctly. Although Mike struggled to find the mileage in part A, he was still able to use good reasoning to answer part D and find the pace. Maya started to reason multiplicatively and no longer did computation with incompatible units (e.g., 10 miles minus 2 minutes).

We found it was difficult for students to use multiplicative reasoning in certain contexts. For example, students struggled to understand how equations represented proportional relationships. Initially, to address this difficulty, we used tasks from the *Illustrative Mathematics curriculum* for our lesson activities. We chose activities that developed proportional reasoning as students investigated graphs and equations. We found, however, that some of the activity contexts did not encourage high levels of student engagement. Some activity contexts were then revised to fit the capabilities and interests of our students, which allowed them to willingly focus and participate in an online environment. For example, because students expressed interest in the shrimp-eating and ice cream contexts during our lessons, we continued to base tasks on these contexts rather than the ones in the original curriculum. We recommend that others make similar adaptions as they work with their students. We also recommend that others teaching online employ techniques such as breakout rooms and private chats to allow each process their thoughts individually and as a result ultimately engage more deeply, in small and large group discussions.