An Exploration of Students' Base-Ten Concepts

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Introduction

Background

Research suggests that teachers must take multiple measures to promote students' understanding of place value. These measures include but are not limited to:

- Understanding students' thought processes and understanding that sequential counting does not necessarily aid in understanding numerical values (Carpenter, Fennema & Franke 1996).
- Moving students' thinking beyond procedural fluency to conceptual understandings (Kilpatrick, Swafford, & Findell, 2001).
- Fostering students' abilities to work multiplicatively and to understand the processes related to grouping by tens (Carpenter et al, 2015).

Goal

The goal of this research was to analyze the ways in which students engaged with and thought about place value content. We sought to design lessons that would move each student along his or her own unique individual learning trajectory to understanding place value.

Research Question

What are the characteristics of the learning trajectories students follow as they engage in a sequence of tasks designed to build place value understanding?

Theoretical Framework

Five Strands of Mathematical Proficiency

(Kilpatrick, Swafford, & Findell, 2001, p. 116)

Conceptual Understanding	Comprehension of mathematical concepts, operations, and relations
Procedural Fluency	Skill in carrying out procedures flexibly, accurately, efficiently and appropriately.
Strategic Competence	Ability to formulate, represent, and solve mathematical problems.
Adaptive Reasoning	Capacity for logical thought, reflection, explanation, and justification.
Productive Disposition	Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with belief in diligence and one's own efficacy.

Conceptual Understanding is strengthened as students represent numerical values and operations pictorially (Suh, 2007).

Procedural Fluency is strengthened as students combine numbers, naturally begin to group by ten, and look for efficient strategies to do so (Wenrick, Behrend & Mohs, 2013).

Strategic Competence is built as students share diverse strategies for solving grouping and combining problems with one another during classroom discussion (Carpenter et al., 2015).

Adaptive Reasoning is strengthened in a classroom that allows students to reflect upon their own thinking and to question the thinking of others (Suh, 2007).

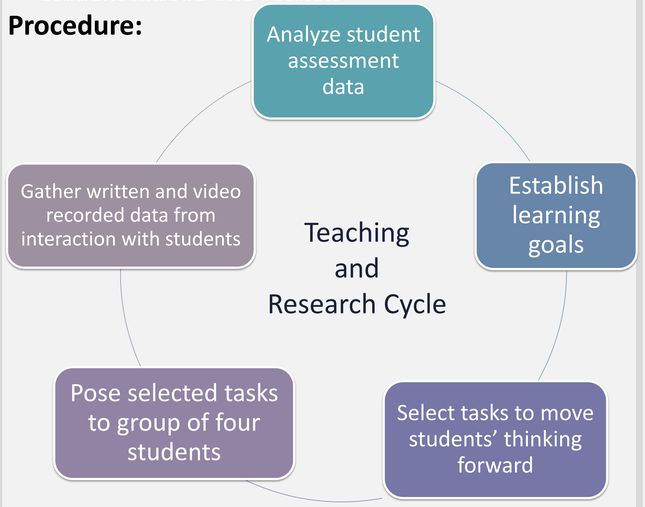
Productive Disposition is strengthened as teachers plan contextualized lessons referring to the base ten system. Problems set in contexts familiar to students can motivate them to bring the full power of their reasoning to the task (Suh, 2007).

Methodology

Participants:

- Four students- three male and one female
- Student pseudonyms: Katie, Gavin, Michael, and Eli
- Transitioning from Grade 3 to Grade 4
- Each participated in a pre- and post- assessment interview (30 minutes each)
- Three students participated in all sessions, one student missed one session

Procedure:

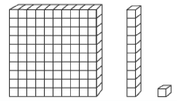


This cycle was repeated seven times. Researchers analyzed data on students' mathematical thinking by reviewing lesson video recordings and transcripts. The five strands of mathematical proficiency were used as a qualitative data analysis framework. Findings about students' strengths and weaknesses in each strand were used to plan each subsequent lesson.

Methodology

Sample Interview Questions

1. What is the name and value of each block?
2. How does the tens rod compare to the unit block?
3. How does the hundreds flat compare to the other blocks?
4. What would the next block look like and how do you know?



Task Purpose: *Checking students' knowledge of names and values of representations of the base-ten system. Comparing place values and the concept of ten times greater and ten times smaller.*

What does each digit represent in the number 212, draw a picture using a base ten model.

Task Purpose: *"Understand that three digits of a three-digit number represent amounts of hundreds, tens, and ones." (CCSSI, 2010) Representing a three digit number and use justification.*

Mentally add 34 + 10. Explain your thinking. Can you add 30?

Task Purpose: *Elicit the thinking strategies that students use for adding 10 to a number as well as strategies that they use for adding multiples of ten.*

Use manipulatives to show 911-24.
(initial question on grade level)

Use manipulatives to show 91-24.
(below grade level)

Solve 1,100,000 – 234,562 and explain your thinking.
(above grade level)

Empirical Teaching and Learning Trajectory:

Initial Assessment Results

Students showed varying strengths in procedural fluency, but all lacked conceptual understanding to some degree. Students performed traditional addition and subtraction algorithms correctly but were unable to explain their thinking with drawings or manipulatives.

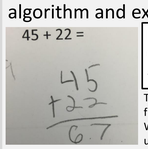
Task: Use manipulative to show 213.

T: Okay so what did you do?
Michael: A one, and then three tens and then two hundred. (picks up a unit cube, three tens rods and two hundreds flats)

Referencing base-ten blocks

Michael showed a lack of conceptual understanding when showing and justifying the representation of 213 by giving a representation of number 231 instead.

Task: Solve the following problem with an algorithm and explain your thinking.

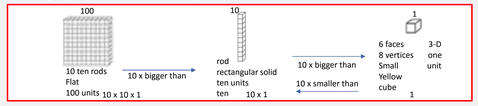


MB: I did up and down. I did five, I did five plus two equal seven, and four plus two equal six.

The student demonstrates strength in procedural fluency by correctly solving the standard algorithm. When explaining the student did not show understanding of the value in the digit 4 in 40 and the digit 2 in 20.

Base-Ten Representation (Week 1-Week 2)

Beginning lessons focused on students' understandings of base-ten blocks. Students were asked to compare blocks, look for patterns, predict the next block and record these results on large chart paper in the correct place value order.



Students had trouble comparing different base 10 blocks to one another. Lack of understanding of "ten times greater" was a barrier, as shown in the dialog excerpt below:

Teacher: How many (rods in a hundreds flat)?
All students: Ten.
Teacher: How many times bigger (is the hundreds flat than the tens rod)?
Katie: Nine.

Together students agreed that the hundreds flat was nine times larger than the tens rod. This showed students' lack of knowledge of multiplicative comparison, specifically in terms of the base-ten number system.

Multiplicative Comparison (Week 3-Week 5)

Task: Researchers placed one Starburst on the table and asked students to make it five times greater.

Teacher: Tell us what you think in your head.
Eli: Um she had five times bigger so I just put five on the table.

Students were successful with problems that only involved creating multiple copies of one object.
To assess students' abilities to make copies of multiple objects, we gave students problems that involved making a group of more than one Starburst a given amount greater. Three out of four students failed to correctly complete these tasks. For example, when we gave them a pile of three Starburst and asked them to make it four times bigger, they simply put down three more Starburst.
This showed us that students were still thinking additively about the situations, as they did with the base ten blocks earlier in the study.

Grouping by Tens (Week 5 – Week 8)

Students demonstrated memorization of procedures but did not show understanding of the tasks given, therefore these lessons were designed to focus on natural grouping by ten to discover place value.

Task: Donnie boxes his donuts in groups of ten. If Donnie has five full boxes and three leftover donuts, how many donuts does he have altogether?

Eli: I counted by um tens that I added up the five and fifty. Five, ten, fifteen, twenty…wait, wait, ten, twenty, thirty, forty fifty. And then I got um fifty then I added that extra three, which would equal fifty-three.

Task: Represent forty-seven donuts in boxes and left overs.

Michael: (places four boxes in a stack, and seven boxes stacked to the right of the four boxes)

Due to the varying levels of understanding and representations using groups of tens, researchers designed differentiated lessons involving combining numbers. Students were given different numbers of donuts to work with, in accord with the levels of thinking they had displayed.

Examples of differentiated tasks:

Whole Group: Donnie has 47 donuts that he needs to package. How many boxes does Donnie need?
Eli: Donnie has 265 donuts and his friend Ellie has 596 donuts. They want to combine their donuts into one order.
Michael: Donnie has 5 blueberry donuts and his friend Ellie has 9 raspberry donuts. They want to combine their donuts into one order.

Post-Assessment Results

- Eli showed understanding of patterns of ten within place value. During the post-assessment, Eli explained the patterns of ten times larger when moving to the left in place value.
- Michael did not show improvement in conceptual understanding and place value as he continued to grasp for a memorized procedures such as rote skip counting and algorithms when solving problems. In last lessons, Michael proceeded to count boxes of donuts by tens, but also counted single donuts by tens.
- Gavin showed improvement in representation of numbers and operations. Throughout the lessons, Gavin began to perform operations using tens frames.
- Katie showed subtle improvements in procedural fluency as she could now add ten to a number without using an algorithm.

References

Carpenter, T.P., Fennema, E., & Frank, M. L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal, 97* (1), 3-20.
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Reflection and discussion: The most difficult standards to attain were those that involved conceptualizing abstract and build their conceptual understanding in the process. numerical values and operations in the base-ten system. Students should use representations to combine numbers in order to naturally group by tens before memorizing algorithms. When students have memorized procedures and have been taught to rely solely on algorithms, they lack the conceptual understanding of numbers and operations that is necessary for mathematical proficiency. By conceptualizing and contextualizing quantities and grouping by ten, students see place value concepts as less abstract and more concrete.