The Thirty-Sixth Annual
Eastern Shore High School Mathematics Competition

November 14, 2019

Team Contest Exam

Instructions

Answer as many questions as possible in the time provided. To receive full credit for a correct solution, show all work and provide a clearly written explanation. Solutions will be judged based on correctness, completeness and clarity. (Little credit, if any, will be given for a solution consisting of just a number or a single sentence.) Calculators are allowed **only** on the team contest exam.

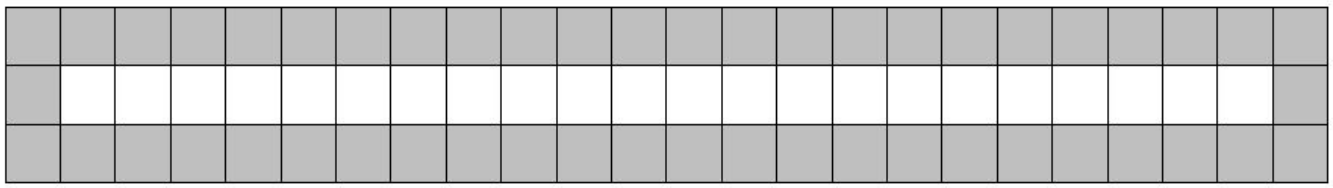
All work and answers must be written on the provided sheets of plain white paper. Use only one side of each sheet of paper, and start each new problem on a new sheet of paper. Write the name of the school which you are representing at the top of each sheet that you turn in for scoring.

At the start of the team round, your team will receive a copy of only Problem 1. Your team must submit a response to Problem 1 within the first 15 minutes of the team round time interval.

When you submit your response for Problem 1, you will receive a copy of Problem 2 and a copy of Problem 3. Your team will then have the time remaining in the team round to complete a response for each problem.

Note: if your team completes Problem 1 before the end of the allotted time, you may submit it and receive copies of Problem 2 and Problem 3 in advance.

1. The rectangle shown below contains exactly 72 small congruent "unit squares" arranged in a 3×24 array. When all of the border unit squares of the 3×24 rectangle are shaded (as shown), 50 of 72 unit squares are shaded and 22 are unshaded.



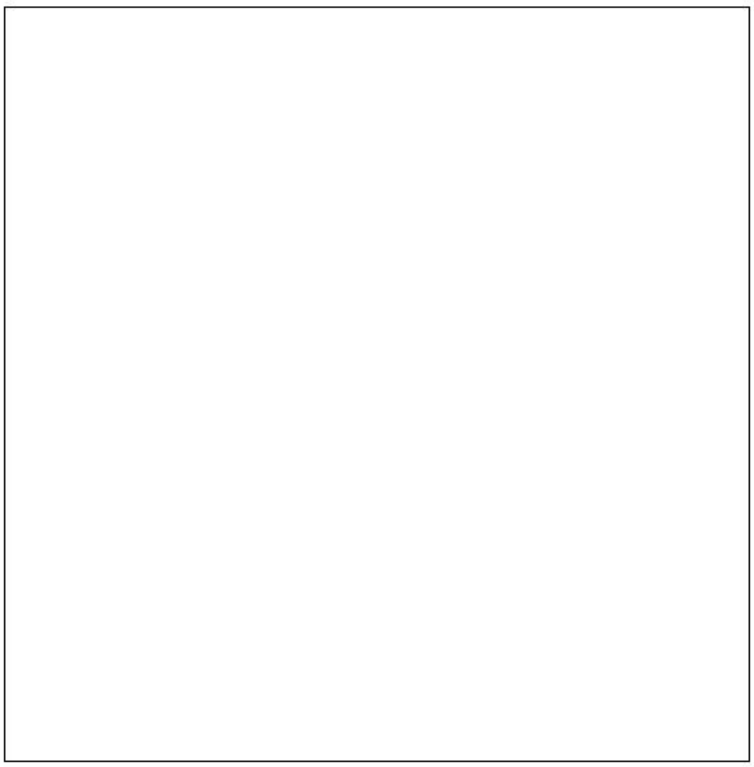
Find a rectangle such that when all of the border unit squares are shaded, the number of unit squares that are shaded is equal to the number of unit squares that are unshaded. The rectangle cannot contain more than 100 square units and the side lengths must be positive integers.

State the dimensions of the rectangle and provide a clear explanation of your solution procedure.

PLEASE NOTE THAT QUESTION 2 IS PRINTED ON BOTH SIDES OF THIS PAGE.

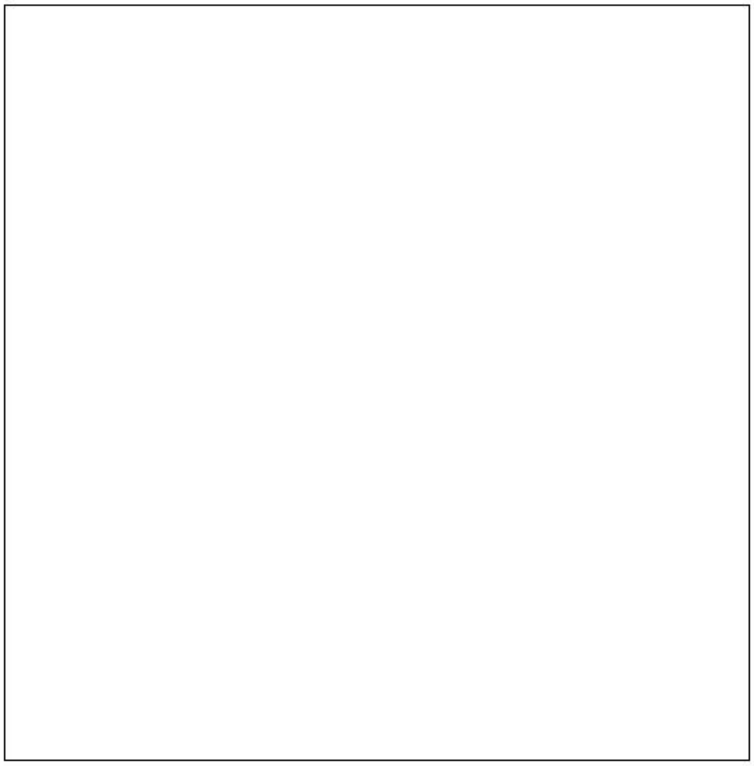
2. In each of the following tasks, all cuts are made perpendicular to opposite sides of the square being cut.

   Part 1: A 4-unit square can be cut into exactly 7 square pieces with no part of the original 4-unit square left over. However, not all of the squares are congruent. Provide a sketch of how the 4-unit square shown below can be cut into exactly 7 square pieces with none of the original 4-unit square left over AND provide a summary of the dimensions of the 7 smaller squares.



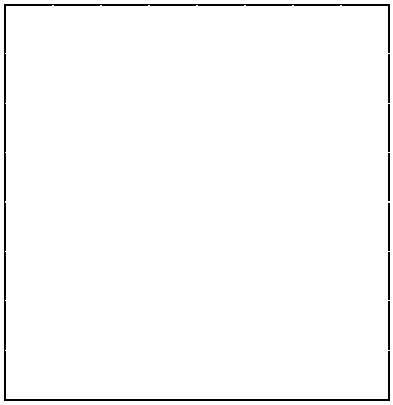
Summary:

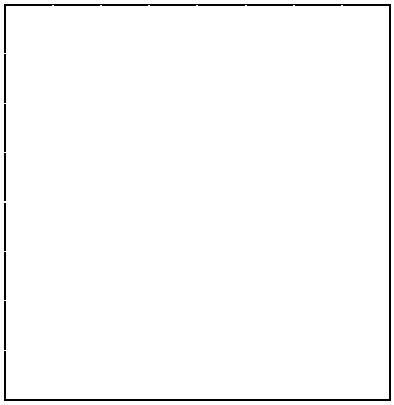
Part 2: A 4-unit square can be cut into exactly 11 square pieces with no part of the original 4-unit square left over. However, not all of the squares are congruent. Provide a sketch of how the 4-unit square shown below can be cut into exactly 11 square pieces with none of the original 4-unit square left over AND provide a summary of the dimensions of the 11 smaller squares.

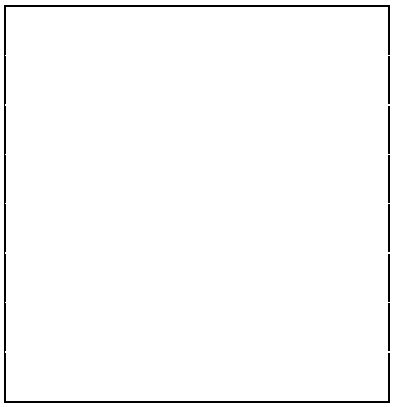


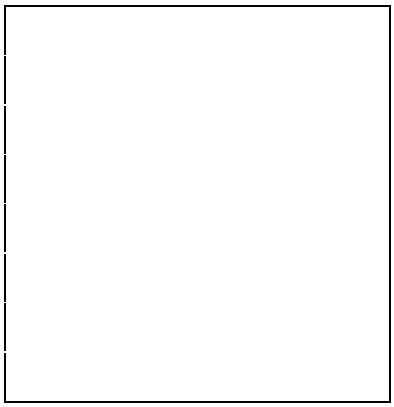
Summary:

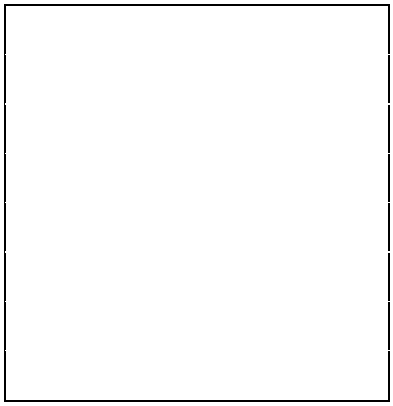
Scrap paper. ***No credit*** *will be given for answers provided on this paper.*

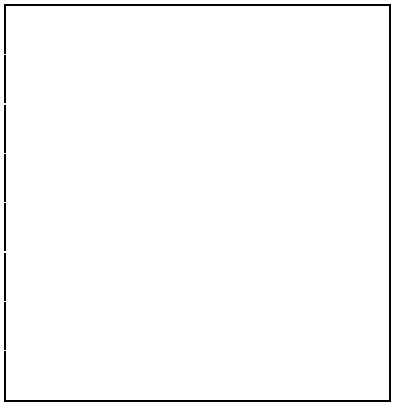






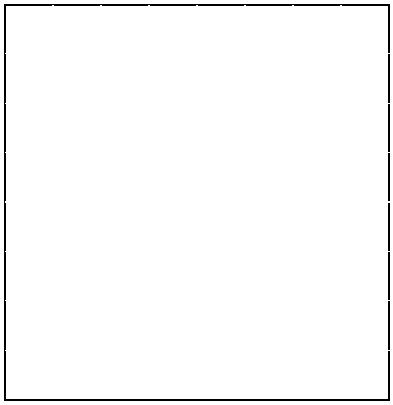


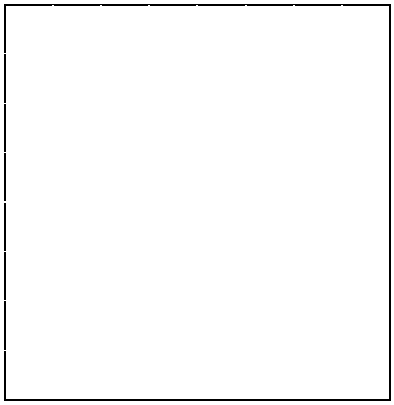


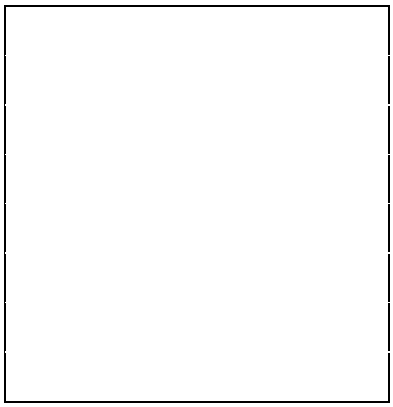


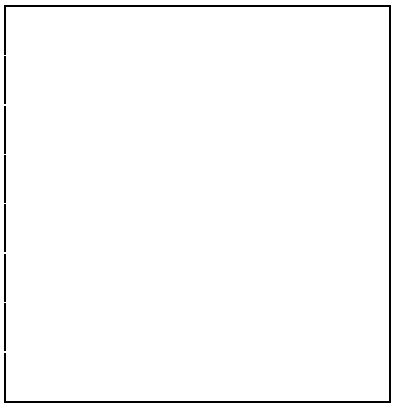
additional squares on the other side →

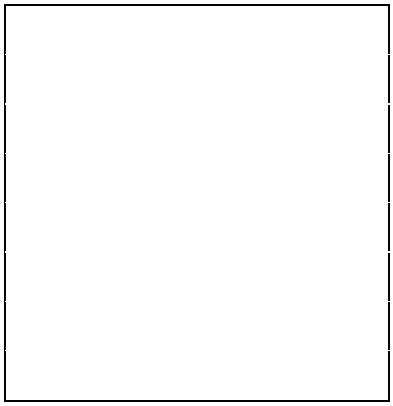
Scrap paper. ***No credit*** *will be given for answers provided on this paper.*

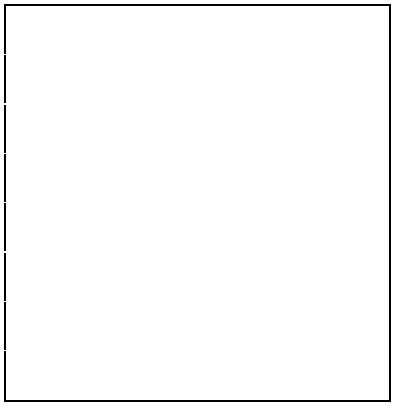












additional squares on the other side →

3. A bag contains 24 number tiles. Twelve of the tiles are numbered "9" and twelve of the tiles are numbered "10." Assume you remove K tiles, $K \in \{2, 3, ..., 24\}$, from the bag and then determine the respective "number tile total." i.e., the sum of all of the numbers on the tiles that you have removed from the bag.

   (a) What is the third largest "number tile total" that CAN BE FORMED with those tiles? Clearly state how many of each type of number tile that was drawn for your solution. Reminder: the "number tile total" is the sum of all of the number tiles that are removed from the bag.

   (b) Using the 24 tiles in the bag what is the largest "number tile total" that CANNOT BE FORMED that is smaller than the largest "number tile total" that can be formed? Clearly explain the reasoning for your solution. Reminder: the "number tile total" is the sum of all of the number tiles that are removed from the bag.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200
201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220
221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260